Money, the existence of which in a modern economy is usually taken for granted, performs three main functions. First, it is a unit of account, second it is a universally accepted means of payment, and thirdly, it is a store of value.

While in models without money one can only analyze the determination of real variables, such as the quantities of goods and services produced and consumed, and their relative prices, in models with money one can also determine nominal variables such as the price level, nominal income, the level of nominal wages, nominal interest rates and inflation. These nominal variables are expressed in terms of money.

In this chapter we focus on the money market in order to analyze the demand for money and the determination of nominal variables such as the price level, nominal interest rates and inflation.

Monetary conditions in modern economies are determined by central banks. The central bank may affect, through a variety of policy instruments at its disposal, both the quantity of bank notes (and coins) in circulation, and, indirectly, the amount of deposits in commercial banks, which are also part of the money supply. Alternatively, a central bank may follow an interest rate rule, intervening in the money market and pegging nominal interest rates. In this case, the stock of money in the economy is determined by the demand for money, and the money supply adapts to demand in order to achieve the goal of the central bank regarding the nominal interest rate.

The demand for money depends on three main factors.

The first factor is the price level. The higher the level of prices, the higher will be the amount of money that households and firms will want to hold for their current and future transactions. The demand for money is usually assumed to be proportional to the price level. This demand stems from the roles of money as both a unit of account and a means of payments.

The second factor is the volume of transactions, usually measured by aggregate real output and income. When the volume of transactions increases, households and firms will need more money to carry out their increased transactions. This determines the demand for real money balances, and stems from the role of money as a means of payments.

The third factor is the level of nominal interest rates. Banknotes pay no interest. On the other hand, demand deposits and current accounts in commercial banks, even when they pay interest, yield a very low return compared with the yields of less liquid assets such as time deposits, treasury bills and bonds. In what follows we shall maintain the assumption that money pays no interest. Thus, as interest rates rise, households and firms will want to hold a smaller part of their assets in the form of
money, compared to interest yielding assets such as time deposits, bonds, or other less liquid assets. Consequently, the demand for money will depend negatively on the level of nominal interest rates.

In this chapter, we first review the basic functions of money and the factors that determine the demand for and supply of money. We analyze the concept of short run equilibrium in the money market, assuming that the central bank follows a policy of either targeting the money supply or pegging nominal interest rates, and also define the notion of the long-run neutrality of money.

We then focus on a number of dynamic general economic equilibrium models with money, we analyze the determination of the price level and nominal interest rates and refer to the long relationship between the money supply, the price level and inflation.

Finally we examine the fiscal incentive for increasing the money supply and its effects on inflation. The most important motive for sustained large increases in the money supply by governments has been the incentive to finance government expenditure that could not be financed by other methods, such as additional taxes or government bonds. This source of revenue for the government is called seigniorage. The main cause of all episodes of sustained high inflation or even hyperinflation, has been the need of governments to use their privilege of printing money, in order to obtain seigniorage.

We examine both the case in which the maximum income from seigniorage in equilibrium is adequate for the financing needs of a government, a situation which can result in an equilibrium with high inflation, as well as the case in which the maximum revenues from seigniorage are not sufficient for the financing needs of the government, which can lead to hyperinflation, which is a disequilibrium phenomenon.

10.1 The Functions of Money

What are the functions of money in an economy, and why do households and firms hold money when there are other assets that pay interest? The answer is that money performs three important functions.

First, money is a unit of account. In a monetary economy all prices are determined and quoted in terms of the monetary unit. Otherwise, economic agents would have to calculate all the relative prices of goods and services in order to conduct their transactions. For example, in an economy with \( N \) goods plus money, there are \( N \) money prices. Without money, economic agents would need to calculate \( N(N+1)/2 \) relative prices in order to make their transactions. As the number of goods and services increases, the number of relative prices to be calculated grows exponentially. For example, if there are 5 goods and services, there are five money prices, and 15 relative prices of goods between them. With 10 goods and services, there are 10 money prices, and and 55 relative prices of all goods and services. With 1000 goods and services, there are 1000 money prices, and 500,500 relative prices between goods and prices. Money therefore helps to simplify the calculation of prices and values, and thus facilitates economic transactions through its unit of account function.

Secondly, money is a generally accepted means of payment. Being accepted by all, money greatly facilitates economic transactions and drastically reduces their costs. Without money, in order to complete a transaction the seller of a product or service would have to find a buyer who would be prepared to offer in return another good or service that the seller wishes to acquire. This requires
that there is a double coincidence of wants in all economic transactions. Transactions of this kind are called barter, which implies huge costs on the part of economic agents in order to find suitable counter-parties to their transactions. A modern economy would immediately cease functioning if there was not a generally accepted medium of exchange and payments, because transaction costs would become prohibitive.

Third, money is a store of value, i.e. a means of holding wealth, and is indeed the asset that is characterized by greater liquidity, as it can be used directly for payments for the acquisition of goods and services. This is a key feature of money, because if money were not a store of value, and lost its value quickly, it would not be generally accepted as a means of payments either. Then again, since money is the only store of value which is also a means of payments, by definition it is the most liquid store of value. However, as a means of holding wealth, money has the weakness that it does not pay interest, unlike other less liquid assets.

All three functions of money as a unit of account, a means of payments and a store of value determine its social role in an economy, and help explain why households and firms attach such great importance to money.

We next turn to the determinants of the supply and the demand for money?

10.2 The Supply of Money and Central Banks

We define as money the sum of banknotes, coins and deposits in current accounts in commercial banks held by households and firms.

This definition of money supply is usually known as \( M1 \). It emphasizes the more liquid assets of households and firms, which usually do not yield interest. However, there are broader definitions of the money supply, that include less liquid assets such as time deposits and other less liquid deposits and securities.

Deposits of credit institutions and other institutions participating in the interbank market and the foreign exchange market are not considered as part of the money supply. These deposits are not used for the transactions of the general public.

The aggregate money supply of an economy, whether narrow or broad, is influenced by central banks. Central banks are public institutions that manage a state’s money supply, interest rates and regulate the commercial banking system. In most countries the central bank possesses a monopoly on printing notes, and minting coins, which serve as the state’s legal tender. In addition, central banks usually act as lenders of last resort to the banking system and, in many cases, the government. Central banks can thus directly determine the circulation of notes (and coins) and indirectly the amount of deposits in commercial banks.

Prior to the 17th century money was mostly commodity money, typically gold, silver or bronze coins. Bronze coins were used for low denomination transactions. However, promises to pay (bank notes) circulated widely, and accepted as money, at least five hundred years earlier in both Europe and Asia.
As the first public bank to “offer accounts not directly convertible to coin”, the Bank of Amsterdam, established in 1609, is considered to be the precursor to modern central banks. The central bank of Sweden (“Sveriges Riksbank” or simply “Riksbanken”) was founded in Stockholm from the remains of the failed bank Stockholms Banco in 1664 and answered to parliament (“Riksdag of the Estates”). One role of the Swedish central bank was lending money to the government. The establishment of the Bank of England, the model on which most modern central banks have been based, was devised by Charles Montagu, 1st Earl of Halifax, in 1694. He proposed a loan of £1.2M to the government; in return the subscribers would be incorporated as The Governor and Company of the Bank of England, with long-term banking privileges, including the issue of notes. The Royal Charter was granted on 27 July through the passage of the Tonnage Act 1694.

Although some would point to the 1694 establishment of the Bank of England as the origin of central banking, the Bank of England did not originally have the same functions as a modern central bank, namely, to regulate the value of the national currency, to finance the government, to be the sole authorized distributor of banknotes, and to function as a “lender of last resort” to banks suffering a liquidity crisis. The modern central bank evolved slowly through the 18th and 19th centuries to reach its current form.

The determination of the money supply by central banks is not a simple process. It depends on the rules under which the central bank participates in money and asset markets and regulates the financial system, on its relations with the government, and on the goals envisaged in its charter.

The main goals of a central bank are the control of inflation, the stability of the financial system, and in some cases, the support of the general economic policies of the government. In what follows we shall ignore many of the institutional details that relate to how a central bank operates, and will make two alternative simple assumptions.

First, we shall assume that the central bank has full control of the money supply. This is an assumption with a long history in macroeconomic analysis, although not particularly realistic, as central banks have imperfect control over the money supply.

Alternatively we shall assume that the central bank follows a policy of determining (pegging) the nominal interest and committing to providing unlimited credit to households, businesses and commercial banks at this rate. This policy of interest rate determination, which many find as a more realistic description of how central banks operate in modern economies, means that the money supply is determined by the demand for money, at the pegged nominal interest rate.

10.3 The Demand for Money

The demand for money by households and firms depends on three main factors.

The first factor is the price level. The higher the level of prices, the higher will be the amount of money that households and firms would want to hold for their current and future transactions. If for example the price level were to double, for a household or a firm to buy the same amount of goods and services, there will be a need to use twice as much money. The demand for money is thus usually assumed to be proportional to the price level.

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1 See Goodhart (1988), among others, for the evolution of central banks.
The second factor is the volume of transactions. When the volume of transactions, usually measured by aggregate real output, increases, households and firms will need more money to carry out their increased transactions.

The third factor is the level of interest rates. Banknotes pay no interest. On the other hand, demand deposits and current accounts, even when they pay interest, pay a very low rate compared to the yields of less liquid assets such as time deposits, treasury bills or bonds. As interest rates rise, households and firms would want to hold a smaller part of their assets in low (or zero) yielding money, in relation to time deposits, securities or other less liquid assets that pay interest. Consequently, the demand for money will depend negatively on the nominal interest rate, as the nominal interest rate measures the opportunity cost of holding money.

The money demand function is usually written as,

\[ M^d = P \times m(Y,i) \]  \hspace{1cm} (10.1)

where \( M^d \) denotes nominal money demand, \( P \) the price level, \( Y \) real aggregate income (GDP) and \( i \) the nominal interest rate. \( m \) is a function increasing in real aggregate income and decreasing in the nominal interest rate. The demand for money is proportional to the price level, in the sense that an increase in the price level requires an increase in the quantity of money by the same proportion, in order to complete the same number of transactions.\(^2\)

The demand for money can thus be written as,

\[ \frac{M^d}{P} = m(Y,i) \]  \hspace{1cm} (10.2)

where (10.2) determines the demand for real money balances.

*Real money demand* as a function of the *nominal interest rate* is depicted in Figure 10.1. The relationship between real money demand and the nominal interest rate is negative, because holding money becomes more expensive as interest rates rise, since money does not pay interest. Therefore, households and firms reduce the amount of money holdings and increase holdings of securities and other interest yielding assets.

The position of the money demand function in Figure 10.1 depends on the level of real income, which determines the volume of transactions in goods and services. Increasing real income for given nominal interest rates, will increase the demand for money as it will increase the amount of money required by households and firms to carry out their increased transactions. The money demand curve will move to the right, as shown in Figure 10.2.

\(^2\) The classic partial equilibrium models of money demand for transactions purposes, that result in an equation such as (10.1), are due to Baumol (1952) and Tobin (1956). The restatement of the quantity theory of money, by Friedman (1956), also results in money demand functions of the form of (10.1), with the additional assumption that the elasticity of the demand for real money balances with respect to real income is also equal to unity.
Therefore, we have argued that households and firms hold money because of the liquidity it provides. How much money households and firms wish to hold depends proportionately on the price level. The demand for money is not a demand for a certain amount of nominal money, but demand for a certain amount of purchasing power. This demand depends positively on the volume of economic transactions (as measured by aggregate real income) and negatively on the opportunity cost of holding money (as measured by the nominal interest rate).

10.4 Nominal Interest Rates and Short Run Equilibrium in the Money Market

The equilibrium condition in the money market is for the money supply to be equal to money demand.

This implies,

\[ \frac{M^s}{P} = \frac{M^d}{P} = m(Y, i) \]

(10.3)

Short run equilibrium in the money market is depicted in Figure 10.3. In Figure 10.3 we assume that the central bank fixes the money supply at some level. We also assume that aggregate real income and the price level are given. The money market equilibrates at the nominal interest rate at which, for given aggregate real income and the price level, the demand for money becomes equal to the supply of money.

As shown in Figure 10.4, an increase in the money supply creates a reduction in the nominal interest rate. An increase in money supply creates excess liquidity in the domestic money market. Households and firms shift this excess liquidity in interest-bearing assets, raising their prices and reducing their yield. This reduces the level of nominal interest rates. In the new equilibrium, given the price level and real income, households and firms voluntarily hold the increased supply of money, as the opportunity cost of holding money, i.e. the nominal interest rate, has fallen.

The negative short-term effect of the money supply on the level of nominal interest rates is often referred to as the liquidity effect. The more liquidity the central bank injects into the money market, in the form of increasing the money supply, the lower the nominal interest rate. Conversely, a decrease in the money supply would reduce liquidity, and cause an increase in the nominal interest rate.

In Figure 10.5, we examine the impact of an increase in money demand. This can be either autonomous (increased demand for liquidity on the part of households and firms), or due to an increase in real income. The last case is the one that we examine in Figure 10.5.

An increase in real income from \( Y_1 \) to \( Y_2 \) raises money demand, because of the increased transactions that have to be financed. Given the money supply and the price level, this creates an excess demand for money for transaction purposes. As households and firms try to move out of interest yielding assets, by liquidating bonds and other interesting yielding assets in order to acquire greater liquidity for transaction purposes, the prices of these assets fall, leading to higher nominal interest rates. The money market will equilibrate at higher nominal interest rates, which will reduce the excess demand for money arising from the increased transactions.
Similar effects would also apply to the case of an autonomous increase in the price level, or an autonomous increase in liquidity preference from households and firms. Given the nominal money supply, an autonomous increase in the price level reduces real money balances, requiring an increase in the nominal interest rate in order for money demand to adjust to the lower supply of real money balances.

Finally, in Figure 10.6 we assume that the central bank follows policy of pegging the nominal interest rate, rather than pegging the money supply. The central bank stabilizes the nominal interest rate at the level \( i_0 \). In this case, the amount of money in the economy is determined by money demand. An increase in the price level or real income, or liquidity preference causes an increase in the money supply, because the central bank is prepared to provide unlimited credit at the nominal interest rate \( i_0 \). As shown in Figure 10.6, when the central bank follows a policy of stabilization of the nominal interest rate, an increase in money demand automatically leads to a higher money supply and vice versa.

### 10.5 The Long Run Neutrality of Money

Our analysis so far was based on the simplifying assumption that real income and the price level are exogenously given. For this reason, the only variable that could adjust to equilibrate the money market was the nominal interest rate. This may be realistic in the very short run, as interest rates are generally more flexible than the prices of goods and services, but it is not realistic in the longer term.

In the longer term, the price level also adjusts. We can see the direction of this adjustment by rearranging the equilibrium condition in the money market, and solving (10.3) with respect to the price level. We then get,

\[
P = \frac{M'}{m(Y,i)}
\]  

(10.4) indicates that the price level depends on the money supply, and the two factors that determine the demand for money, i.e. aggregate real income and the nominal interest rate.

The price level may rise if there is an increase in the money supply, a decline in real income, an increase in the nominal interest rate, or some other extraneous factor that autonomously reduces the demand for money.

In order to explain inflation in the long run, i.e. continuous increases in the price level, the focus has to be on continuous increases in the money supply. As we saw in Chapter 6, in the process of balanced growth real incomes grow at a steady rate, while real interest rates are stabilized. Nominal interest rates are equal to the real interest rate plus expected inflation. Thus, in a steady state with constant inflation, the nominal interest rate is also constant.

Expressed differently, in long run equilibrium, aggregate real income and the real interest rate are on their balanced growth paths. With constant inflation, nominal interest rates are also constant. Thus, the factors affecting the demand for money are given, and the level of the money supply...
determines the price level, without affecting the evolution of real variables. This property is called *long-run neutrality* of money.\(^3\)

In order to support this assertion we should be able to prove that the money supply does not affect real output or real interest rates in the long run.

The neutrality of money applies to all static general equilibrium models with flexible prices. The determinants of the level of equilibrium real income, and other real variables, are the available resources, technology, preferences, the functioning of markets, as well as economic institutions that determine total factor productivity and the productivity of specific factors.

In static general equilibrium models real output and income and other real variables do not depend on the money supply. Money is merely a “veil” which covers the economy, simply determining nominal variables such as the price level.

In dynamic general equilibrium models, such as the ones we examined in Chapter 6, we usually distinguish between the “neutrality” and the “super-neutrality” of money.

The “neutrality” of money refers to the effects of a one off change in the money supply, and the “super-neutrality” of money to the effects of the rate of change of the money supply.

The neutrality of money applies to all dynamic general economic equilibrium models with flexible prices.\(^4\)

However, as we saw in Chapter 6, the growth rate of money supply affects inflation and long-term nominal interest rates, and thus affects real money demand.

In a representative household model, the growth rate of the money supply does not affect any other real variable, apart from real money balances. Consequently, it could be argued that the “super-neutrality” of money applies to representative household models. This is the case of the Sidrauski (1967) model we analyzed in Chapter 5.

In overlapping generations models the “super-neutrality” of money does not apply, as the growth rate of the money supply affects savings and the accumulation of capital, and thus all other real

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\(^3\) The long run relationship between the money supply and the price level has been analyzed since the 16th century, on the basis of the quantity theory of money, according to which the quantity of money demanded is proportional to the volume of transactions (aggregate real income) and the price level. Among the first who analyzed the relationship between the money supply and the price level was Copernicus, who, in a memorandum of 1517, used the quantity theory to explain the large increase in the price level in the early 16th century. The quantity theory of money has since been refined by many analysts and economists (Hume 1752, Mill 1848), as an explanation of the determination of the price level. Algebraically, it took two alternative forms (see Humphrey 1984). First, the form of the equation of exchange, \(MV=PY\), where \(V\) is the velocity of money (Newcomb 1885, Fisher 1911). Alternatively, according to the Cambridge School, it took the form of a money demand function, \(M=kPY\), where \(k\) is the percentage of income held in the form of money (Pigou 1917, Keynes, 1923). After World War II, the quantity theory of money was restated by Milton Friedman (see Friedman 1956, Friedman and Schwartz 1963), who also stressed the role of nominal interest rates and expected inflation. Alternative partial equilibrium model of money demand were developed by Baumol (1952) and Tobin (1956).

\(^4\) The neutrality of money in the long run holds true even in models that are not characterized by super-neutrality, in the sense that the growth rate of the money supply affects real variables. Such is the Weil (1987, 1991) overlapping generations model examined in Chapter 6.
variables on the steady growth path. This is the case with the Weil (1987, 1991) model, also analyzed in Chapter 5.

An alternative way to think about the neutrality of money, is to consider what would be the impact of a very radical change in the money supply. Such radical changes take place in times of monetary reforms. A number of such historical examples exist, which suggest that, after a monetary reform, the price level adjusts immediately to the new monetary standard.\(^5\)

Gradual increases in the money supply in the long run have effects similar to such monetary reforms. The tripling of the money supply in a decade, in the long run has the same effect as a monetary reform in which a currency unit is replaced with three units of a “new” currency.

Thus, while a short-term change in the money supply can cause equilibrating changes in nominal interest rates, in the longer term, what adjusts in order to equilibrate the money market is the price level. Nominal interest rates return to their long-run equilibrium, determined by the real interest rate plus expected long run inflation.

**10.6 Money and the Price Level in Dynamic General Equilibrium Models**

In order to examine the determinants of money demand, the role of money, but also the long-run neutrality of money, we will analyze a series of dynamic general equilibrium models, in which prices are flexible and the demand for money results from the optimizing behavior of households and firms.

As we shall see, the long-term neutrality of money is a property of all the models examined, although these models have different properties regarding the role of money and the operation of the money market, the determination of the price level, the liquidity effect and the implications of interest rate rules.

**10.6.1 The Samuelson Overlapping Generations Model**

We shall start with the overlapping generations model of Samuelson (1958), in which the demand for money arises only from its role as a store of value.

We assume that the economy consists of successive generations of households, each of which lives for two periods. Every household has exogenous income \(y_1\) in the first period of life and \(y_2\) in the second period of life. This income is in the form of a non storable good, which cannot be transferred from period to period. The only non-perishable commodity is money, which can be used as a means of holding wealth.

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\(^5\) For example, in May 1954, there was a radical monetary reform in Greece. A new drachma was created, which amounted to 1,000 old drachmas. Essentially this amounted to a direct reduction in the money supply to one thousandth of the old money supply. As one would expect on the basis of (10.4), the price level in Greece fell immediately to one thousandth of the price level before the reform. Nothing else changed, other than the level of prices. Similar monetary reforms, involving the redefinition of the value of a national currency have taken place in many other countries. Mexico redefined the peso in January 1993, by creating a new peso, equal to 1,000 old pesos. The price level fell to one thousandth of the old price level. Turkey redefined the lira in January 2005, by creating a new turkish lira, equal to 1,000,000 old liras. The price level fell to one millionth of the previous price level. Argentina and Brazil have also gone through a number of such monetary reforms. The creation of the euro was also a monetary reform of this nature, as the euro replaced national currencies of different denominations, causing immediate changes in the price level related to the conversion rates to the new currency, for all countries that adopted the euro.
The utility function of the generation born at time $t$ depends on the consumption of goods in the first and second period of her life. Consequently, the household born in period $t$ maximizes the utility function,

$$U_t = u(C_{1t}) + \beta u(C_{2t+1}) \quad (10.5')$$

under the constraints,

$$P_tC_{1t} + M = P_tY_1 \quad (10.6)$$

$$P_{t+1}C_{2t+1} = M + P_{t+1}Y_2 \quad (10.7)$$

$C_t$ is household consumption in the first period of life, $C_2$ consumption in the second period of life, $u$ a concave utility function and $\beta=1/(1+\rho)$ the discount factor, where $\rho$ is the pure rate of time preference. $M$ is the money supply, carried over by the household from the first to its second period of life. The money supply is equal to the savings of households in their first period of life. $P_t$ is the money price of the consumption good in period $t$ and $P_{t+1}$ the money price of the consumption good in period $t+1$.

We will assume that the household utility function is logarithmic, and takes the form,

$$\ln C_{1t} + \beta \ln C_{2t+1} \quad (10.5)$$

From the maximization of (10.5) under the constraints (10.6) and (10.7) it follows that the consumption of the young in period $t$ is determined by,

$$P_tC_{1t} = \frac{1}{1+\beta} (P_tY_1 + P_{t+1}Y_2) \quad (10.6)$$

The old generation in period $t$, those who are in their second period of life, consumes all its current income, plus its savings, i.e. the quantity of money carried over from the previous period.

$$P_tC_{2t} = M + P_tY_2 \quad (10.7)$$

The equilibrium condition in the goods market implies that,

$$C_{1t} + C_{2t} = Y_1 + Y_2 \quad (10.8)$$

From (10.6)-(10.8) it thus follows that,

$$(1+\beta)M = \beta Y_1P_t - Y_2P_{t+1} \quad (10.9)$$

Solving (10.9) for the demand for real money balances,
From (10.10) it follows that, if there exists a constant equilibrium price level $P^*$, this should satisfy,

$$\frac{M}{P} = \frac{1}{1+\beta} \left( \beta Y_i - \frac{P_{rel} Y_2}{P_i} \right)$$

(10.10)

The condition for a positive equilibrium price level, and thus a positive demand for real money balances is that,

$$\frac{\beta Y_i}{Y_2} > 1$$

(10.12)

The demand for money, and hence the price level, will be positive only if the discounted first period income of households exceeds second period income. It is only then that savings, and hence money demand, will be positive.

The Samuelson model has a striking implication. Money improves welfare, because it allows households to engage in inter-temporal trade and smooth consumption over time. In the absence of money, consumption in each period would have to be equal to current income for all generations. This equilibrium is clearly suboptimal compared with the equilibrium of a monetary economy which allows for consumption smoothing.

In order to examine the dynamic adjustment of the price level, we can substitute (10.11) in (10.9). The resulting adjustment equation for the price level takes the form,

$$P_{rel} - P^* = \frac{\beta Y_i}{Y_2} (P_i - P^*)$$

(10.13)

Since the price level is a non predetermined variable, the condition for the stability of the dynamic adjustment to the equilibrium price level $P^*$ is (10.12), i.e. that the root of the difference equation (10.13) is greater than one. Consequently, the condition for the existence of a positive equilibrium price level coincides with the condition for the stability of the equilibrium. If (10.12) is satisfied, then a positive equilibrium price level exists, and in addition the equilibrium is a saddle point, i.e. dynamically stable.

The Samuelson model of overlapping generations is one of the first dynamic general equilibrium models that generate a positive demand for money as a store of value. The neutrality of money follows immediately. From equation (10.11), an increase in the money supply $M$ will cause an increase in the equilibrium price level $P^*$ by the same percentage. Moreover, in this model, since the price level is a non predetermined variable, the increase in the price level would happen immediately.

However, the Samuelson model also has a number of weaknesses as a model of money demand.
Its first weakness is that the equilibrium we have just described, which entails a positive demand for money, is not unique. There is a second, suboptimal, equilibrium, with zero money demand. Thus the demand for money in this model is extremely fragile. To examine this issue, we can divide both sides of (10.9) by \(M\), and solve the resulting equation with respect to \(M/P_{t+1}\). It follows that,

\[
\frac{M}{P_{t+1}} = \frac{M}{P_t} \beta Y_1 - (1 + \beta) \left( \frac{M}{P_t} \right) Y_2
\]

(10.14)

From (10.14) it follows that there are two equilibria for money demand. One is (10.11), and the second is the zero solution,

\[
\frac{M}{P_{**}} = 0
\]

(10.15)

The equilibrium with a price level \(P^*\) is locally stable and well defined, but the system is globally indeterminate, because there are is an infinite number of adjustment paths that, starting with a price level above \(P^*\), converge to the price level \(P^{**}\), that is infinity. This global indeterminacy is analyzed in Figure 10.7.

However, it is worth mentioning that this global indeterminacy does not arise if the income of the second period is equal to zero. In this case, that is if the exogenous household income only occurs during the first period of life, (10.10) turns into,

\[
\frac{M}{P_t} = \beta \frac{Y_1}{1 + \beta}
\]

(10.10')

(10.10’) implies a unique equilibrium for the price level.

A second weakness of this model is that there is no alternative store of value. The only way to save in this model is by holding money. However, if there is an alternative asset which pays interest, for example bonds or capital, then money would be ostracized from this economy, because its only role is as a store of value, and money does not pay interest.

There are two categories of alternative dynamic general equilibrium models which generate a positive demand for money, without the weaknesses of the Samuelson overlapping generations model. These two categories, which we alluded to in Chapter 5, are models in which money enters the utility function of households (money in the utility function models), and models in which economic transactions can only take place through the mediation of money (cash in advance models).

### 10.6.2 Money in the Utility Function of a Representative Household

We have already introduced this class of models, in the context of money and growth models in Chapter 5. This class of models originates with Patinkin (1956), Sidrauski (1967) and Brock (1974, 1975). Unlike the model of Samuelson, in this class of models money can coexist with interest yielding assets, such as bonds, in a dynamic general equilibrium.
We will focus on money demand in an economy in which, as in the Samuelson model, real income is exogenous and there is no capital.

There is a representative household with an inter-temporal utility function of the form,

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s, \frac{M_s}{P_s})$$

(10.16)

where $C$ is real consumption of goods and services, $M$ is the quantity of nominal money balances held by the household, $P$ is the price level, and $u$ is a concave periodic utility function, which is homogeneous of degree one in its two arguments, consumption and real money balances.

The representative household maximizes its inter-temporal utility function under the sequence of budget constraints,

$$C_s + \frac{M_s}{P_s} + \frac{B_s}{P_s} = Y_s - T_s + \frac{M_{s-1}}{P_s} + \frac{(1+i_{s-1})B_{s-1}}{P_s}$$

(10.17)

where, $Y$ is the real income of the household, assumed exogenous, $T$ per capita taxes net of transfers, $B$ the nominal value of bonds held by the household, and $i$ the nominal interest rate.

The Lagrangian corresponding to this problem can be written as,

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \left( u(C_s, \frac{M_s}{P_s}) + \lambda_s \left( \frac{M_{s-1}}{P_s} + \frac{(1+i_{s-1})B_{s-1}}{P_s} + Y_s - T_s - C_s - \frac{M_s}{P_s} + \frac{B_s}{P_s} \right) \right)$$

(10.18)

where $E_t$ is the mathematical expectations operator, on the basis of information available in period $t$.

The first order conditions for a maximum with respect to $C$, $B$ and $M$ imply,

$$\lambda_s = \frac{\partial u}{\partial C_s}$$

(10.19)

$$\frac{\lambda_s}{P_s} = \beta(1+i_s)E_t \left( \frac{\lambda_{s+1}}{P_{s+1}} \right)$$

(10.20)

$$\frac{\lambda_s}{P_s} = \frac{1}{P_s} \frac{\partial u}{\partial M_s} + \beta E_t \left( \frac{\lambda_{s+1}}{P_{s+1}} \right)$$

(10.21)

These first order conditions have the usual interpretations.

(10.19) is the static first order condition, according to which, the marginal utility of consumption should in any period is equal to the “shadow value” of marginal savings. Essentially, the household should be indifferent at the margin between consumption and savings.
(10.20) is the dynamic first order condition, according to which the total expected real return on savings should be equal to the pure rate of time preference of the household. This can be seen if we take the logarithm of (10.20). We have that,

\[ i_t - E_t (\ln P_{t+1} - \ln P_t) + E_t (\ln \lambda_{t+1} - \ln \lambda_t) = -\ln \beta = \rho \]

(10.20')

The left hand side of (10.20') is the total expected real return on savings, taking into account expected inflation and expected capital gains from a change in \( \lambda \). The right hand side is the pure rate of time preference of the household, as \( \beta = 1/(1+\rho) \).

Finally, (10.21) is the dynamic first order condition according to which the marginal utility of real money balances is equal to the difference of the pure rate of time preference from the expected real return of money, taking into account expected inflation and expected capital gains from a change in \( \lambda \).

From these three first order conditions we can derive the demand for money.

Let us assume, as in Chapter 5, that the periodic utility function takes the form,

\[ u = \ln \left( \gamma C_t \right)^\varepsilon + (1 - \gamma) \left( \frac{M_t}{P_t} \right)^\varepsilon \]

(10.22)

where \( 1/(1-\varepsilon) \) is the elasticity of substitution between consumption and real money balances.

Under this assumption, from the first order conditions (10.19)-(10.21), the demand for money function takes the form,

\[ \frac{M_t}{P_t} = \left( \frac{\gamma i_t}{1 - \gamma \frac{1}{1+i_t}} \right)^{\frac{1}{1-\varepsilon}} C_t \]

(10.23)

The money demand function depends negatively on the nominal interest rate, and positively on total consumption. The negative dependence on the nominal interest rate arises because with higher nominal interest rates the opportunity cost of holding money compared to bonds is higher, and this reduces the demand for money.

As in the model of Samuelson, for given income and consumption, and given the nominal interest rate, a one-off increase in the money supply leads to an increase in the price level by the same percentage. As can be seen from (10.23), the *neutrality of money* holds in this model as well.

10.6.3 Cash in Advance in a Representative Household Model

The basic idea of models in which money is the only means of payment, is that in order to complete any economic transaction, payment must be in money, and in particular cash, which the buyer holds in advance of the completion of the transaction. This idea is due to Clower (1967), and its
integration into general equilibrium models leads to a class of models known as *cash in advance* models.

The restriction that the transaction must be paid with money held in advance, imposes a cost of holding money, because, alternatively, economic agents could hold another asset, such as bonds, which pays interest.

The cash in advance restriction can take several forms, depending on the assumptions made about the sequencing of transactions. A simple traditional way of expressing this constraint is given by,

\[ P_t C_t \leq M_{t-1} \] (10.24)

where \( M_{t-1} \) is the stock of money accumulated until the end of period \( t-1 \). The problem with this version of the constraint is that someone who enters the economy in period \( t \) would not be able to consume at all, since she holds no money.

An alternative hypothesis is that each period consists of two different sub-periods. In the first sub-period agents visit a financial market, say a bank, where they can swap interest bearing assets with money, or borrow cash, and in the second sub-period they deal in markets for goods and services, which are liable to the cash in advance constraint (see Helpman 1981, Lucas 1980, 1982). This allows the following two-part form of the constraint,

\[ A_t = M_t + B_t \] (10.25)

\[ P_t C_t \leq M_t \] (10.26)

where \( A_t \) is the stock of all nominal assets of the household. \( M_t \) is nominal money balances and \( B_t \) the value of nominal bonds held by the household.

In the second sub-period, households also receive their exogenous real income \( Y_t \) and pay their taxes (net of transfers) \( T_t \). As a result, the nominal assets of the household in the beginning of the following period are determined by,

\[ A_{t+1} = M_t + (1+i_t)B_t + P_t(Y_t - T_t - C_t) = (1+i_t)A_t - i_t M_t + P_t(Y_t - T_t - C_t) \] (10.27)

The representative household thus maximizes the inter-temporal utility function,

\[ U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \] (10.28)

under the sequence of budget constraints (10.27) and the cash in advance constraint (10.26).

The Lagrangian is given by,

\[ E_t \sum_{s=t}^{\infty} \beta^{s-t} \left( u(C_s) + \lambda_t \left( M_t - P_t C_t \right) + \xi_t \left( 1+i_t \frac{A_t}{P_t} - (Y_t - T_t - C_t - i_t \frac{M_t}{P_t} - \frac{A_{t+1}}{P_t}) \right) \right) \] (10.29)
where $\nu$ and $\lambda$ are the two Lagrange multipliers.

The first order conditions for a maximum imply that,

$$
\lambda_t + \nu_t = \frac{\partial u}{\partial C_t} = u'(C_t) \tag{10.30}
$$

$$
\frac{\lambda_t}{P_t} = \beta E_t \left( (1 + i_{t+1}) \frac{\lambda_{t+1}}{P_{t+1}} \right) \tag{10.31}
$$

$$
\nu_t = \lambda_t i_t \tag{10.32}
$$

The interpretation of the first order conditions is straightforward.

(10.30) is the static first order condition according to which the optimal consumption equates the marginal utility of consumption with the “shadow value” of savings $\lambda$, plus the shadow value of money $\nu$. The shadow value of money results from the restriction for cash in advance in order to buy consumer goods.

(10.31) is the dynamic first-order condition, according to which the total expected real return on savings, including expected inflation and expected capital gains, should be equal to the pure rate of time preference of the household.

Finally, (10.32) is the static first order condition according to which, the shadow value of money should be equal to the shadow value of savings times the opportunity cost of holding money, which is none other than the nominal rate, since money pays no interest.

Combining (10.30)-(10.32) one gets,

$$
\frac{u'(C_t)}{P_t} = \beta (1 + i_{t+1}) E_t \left( \frac{u'(C_{t+1})}{P_{t+1}} \right) \tag{10.33}
$$

(10.33) is a monetary form of the usual Euler equation for consumption in this model, in which consumption requires money payments in advance.

Assuming logarithmic preferences,

$$
\frac{\partial u}{\partial C_t} = \frac{1}{C_t} \tag{10.34}
$$

Under the assumption of logarithmic preference, (10.33) can be written as,

$$
\frac{1}{P_tC_t} = \beta (1 + i_{t+1}) E_t \left( \frac{1}{P_{t+1}C_{t+1}} \right) \tag{10.35}
$$
The cash in advance constraint implies that,

$$\frac{M_t}{P_t} = C_t$$  \hspace{1cm} (10.36)

(10.36) determines the demand for money in this model. Monetary neutrality holds in this model as well.

Substituting (10.36) in (10.35), we get,

$$\frac{1}{1 + i_{t+1}} = \beta E_t \left( \frac{M_t}{M_{t+1}} \right)$$  \hspace{1cm} (10.37)

The nominal interest rate depends positively on the pure rate of time preference $\rho$, which determines $\beta$, and the expected rate of change of the money supply, which determines expected inflation.

### 10.6.4 Cash in Advance in an Overlapping Generations Model

We finally examine the implications for money demand of a cash in advance constraint in a variant of the Samuelson overlapping generations models. In this model, money functions both as a means of payments and a store of value.\(^6\)

The household born in the beginning of period $t$ lives for two periods, period $t$ and period $t+1$. It receives income $Y_t$ in the first period of life, and consumes in both periods.

The inter-temporal utility function of the household is given by,

$$U_t = \ln C_{1t} + \beta \ln C_{2t+1}$$  \hspace{1cm} (10.38)

In each period of life the household is subject to a cash in advance constraint of the form,

$$P_t C_{1t} \leq M_{1t}, \quad P_{t+1} C_{2t+1} \leq M_{t+1}$$  \hspace{1cm} (10.39)

Total consumption and the money supply in each period are given by,

$$C_t = C_{1t} + C_{2t}, \quad M_t = M_{1t} + M_{2t}$$  \hspace{1cm} (10.40)

Total assets of households are equal to $A$, and we assume that young households are born without assets. As a result, all assets belong to the old households. For simplicity we assume that taxes $T$ are only paid by young households.

---

\(^6\) In Chapter 5 we have already analyzed a Blanchard-Weil overlapping generations growth model with money in the utility function of households, and have already shown that in such a model the super-neutrality of money does not hold.
Given that old households receive no current income, their consumption is equal to their assets. As a result,

\[ C_{2t} = \frac{A_t}{P_t} \] (10.41)

It is worth noting that because of the cash in advance constraint, the old households need to convert their assets into money, in order to purchase consumer goods.

Given that young households hold no assets, they need to borrow and convert their loan into money, in order to finance their consumption. As a result, for young households the following constraints must hold,

\[ M_{1t} = P_t C_{1t}, \quad B_{1t} = -P_t C_{1t} \] (10.42)

As a result, the assets of young households at the end of their first period of life will be equal to,

\[ A_{r+1} = M_{1t} + (1+i_t) B_{1t} + P_t (Y_t - T_t - C_{1t}) = P_t Y_t - T_t - (1+i_t) C_{1t} \] (10.43)

From (10.41) and (10.43) it follows that,

\[ C_{2t+1} = \frac{A_{r+1}}{P_{r+1}} = \frac{P_t (Y_t - T_t - (1+i_t) C_{1t})}{P_{r+1}} \] (10.44)

Introducing (10.44) in the utility function (10.38), we find that young households will choose consumption in their first period of life in order to maximize,

\[ U_t = \ln C_{1t} + \beta \ln \left( \frac{P_t (Y_t - T_t - (1+i_t) C_{1t})}{P_{r+1}} \right) \] (10.45)

From the first order conditions for the maximization of (10.45) it follows that,

\[ C_{1t} = \frac{1}{1+\beta} \frac{Y_t - T_t}{1+i_t} \] (10.46)

From (10.41) and (10.46) aggregate consumption is given by,

\[ C_t = C_{1t} + C_{2t} = \frac{1}{1+\beta} \frac{Y_t - T_t}{1+i_t} + A_t \] (10.47)

From the equilibrium condition in the market for goods and services,

\[ C_t = Y_t = \frac{1}{1+\beta} \frac{Y_t - T_t}{1+i_t} + A_t \] (10.48)
Given the cash in advance constraints it also follows that,\\
\[ M_t = P_t Y_t \]  
\[ (10.49) \]

As a result, we can solve (10.48) and (10.49) for the price level and the nominal interest rate.

The neutrality of money holds in this model as well, given that real income is assumed exogenous.

10.7 Nominal and Real Interest Rates

We now turn to the determinants of the nominal interest rate in the general equilibrium models we have presented.

We will analyze three models. The model with money in the utility function of a representative household, the representative household model with a cash in advance constraint, and finally, the overlapping generations model with a cash in advance constraint. In all three models, money demand is positive, even if consumers have the option of holding interest bearing assets such as bonds.

10.7.1 Money in the Utility Function of a Representative Household

The money demand function in this model is given by (10.23). We shall examine, for reasons of simplification and comparability with the other two models, the case of logarithmic preferences (\( \varepsilon = 0 \)).

With logarithmic preferences, the first order conditions for the maximization of the utility function of the representative household are given by,

\[ \lambda_t = \frac{\gamma}{C_t} \]  
\[ (10.19') \]

\[ \frac{\dot{\lambda}_t}{P_t} = \beta (1 + i_t) E_t \left( \frac{\lambda_{t+1}}{P_{t+1}} \right) \]  
\[ (10.20') \]

\[ \frac{\dot{\lambda}_t}{P_t} = \frac{1 - \gamma}{M_t} + \beta E_t \left( \frac{\lambda_{t+1}}{P_{t+1}} \right) \]  
\[ (10.21') \]

From (10.19’) and (10.20’) it follows that,

\[ \frac{1}{P_t C_t} = \beta (1 + i_t) E_t \left( \frac{1}{P_{t+1} C_{t+1}} \right) \]  
\[ (10.50) \]

(10.50) is the Euler equation for consumption in an economy with money.
From (10.19’) and (10.21’) it follows that,

\[
\frac{1}{P_tC_t} = \frac{1-\gamma}{\gamma} \frac{1}{M_t} + \beta E_t \left( \frac{1}{P_{t+1}C_{t+1}} \right) \tag{10.51}
\]

The solution of (10.51) takes the form,

\[
\frac{1}{P_tC_t} = \frac{1-\gamma}{\gamma} \sum_{s=t}^{\infty} \beta^{s-t} E_t \left( \frac{1}{M_s} \right) \tag{10.52}
\]

Given that \(C_t=Y_t\), which is exogenous, (10.52) determines the equilibrium price level, as a function of expectations about the future evolution of the money supply.

Substituting (10.52) in the money demand equation (10.23) for \(\varepsilon=0\), and solving for the nominal interest rate,

\[
\frac{1+i_t}{i_t} = \frac{\gamma}{1-\gamma} \frac{M_t}{P_tC_t} = \sum_{s=t}^{\infty} \beta^{s-t} E_t \left( \frac{M_s}{M_t} \right) \tag{10.53}
\]

The nominal interest rates is determined by the current money supply and expectations about the future development of the money supply, at a rate of discount that depends on the pure rate of time preference of the household.

Suppose the expected growth rate of the money supply is constant and equal to \(\mu\). From (10.53),

\[
\frac{1+i_t}{i_t} = \sum_{s=t}^{\infty} \beta^{s-t} \left( \frac{1}{1+\mu} \right)^{s-t} = \sum_{s=t}^{\infty} \left( \frac{1}{(1+\rho)(1+\mu)} \right)^{s-t} = \frac{(1+\rho)(1+\mu)}{(1+\rho)(1+\mu) - 1} \tag{10.54}
\]

From (10.54) it follows that,

\[
i_t = (1+\rho)(1+\mu) - 1 = \rho + \mu \tag{10.55}
\]

Consequently, from (10.55), the higher the growth rate of money supply \(\mu\), the higher will be the nominal interest rate \(i\), as the expected future inflation rate will be higher.\(^7\)

It is worth noting that the real equilibrium interest rate in the model is equal to \(\rho\). For \(\mu=0\), (10.55) implies \(i=\rho\). In this case, because the expected future inflation rate is equal to zero, the nominal

\(^7\) (10.55) is a version of the Fisher equation we encountered in Chapter 6. To quote from Fisher (1896), “When prices are rising or falling, money is depreciating or appreciating relative to commodities. Our theory would therefore require high or low interest according as prices are rising or falling, provided we assume that the rate of interest in the commodity standard should not vary.” (p. 58). The rate of interest in the commodity standard is the real interest rate, and rising or falling prices are expected inflation. The Fisher equation was further elaborated in Fisher (1930), where it was made even clearer that Fisher referred to expected inflation.
interest rate equals the equilibrium real interest rate, i.e. the pure rate of time preference of the representative household.

It is worth noting that if $\mu = -\rho/(1 + \rho)$, i.e. if the money supply is reduced at this rate, the nominal interest rate is driven to zero. As we shall see below, a zero nominal interest rate has attractive properties and leads to the optimal money demand by households.

### 10.7.2 Cash in Advance in a Representative Household Model

In the representative household model in which money demand results from the cash in advance constraint, under the assumption of logarithmic preferences, the nominal interest rate is determined by equation (10.37).

Assuming that the growth rate of money supply is equal to $\mu$, (10.37) implies,

$$\frac{1}{1 + i_{t+1}} = \beta \left( \frac{1}{1 + \mu} \right) = \frac{1}{(1 + \rho)(1 + \mu)} \tag{10.56}$$

From (10.56), the nominal interest rate is determined by (10.55), exactly like in the representative household model with money in the utility function.

Consequently, both monetary representative household models, the money in the utility function model and the cash in advance model, have exactly the same predictions concerning the determination of nominal and real interest rates.

### 10.7.3 Cash in Advance in an Overlapping Generations Model

We finally return to the Samuelson overlapping generations model with a cash in advance constraint.

From (10.48) and (10.49) it follows that,

$$1 + i_t = \frac{1}{\frac{1}{1 + \beta} \frac{Y_t - T_t}{Y_t} \frac{M_t}{M_t - A_t}} \tag{10.57}$$

where $M_t - A_t = P_t C_t > 0$.

The nominal interest rate depends only on the current stock of the money supply, and not on its expected future increase.

From (10.57) it follows that,

$$\frac{\partial i_t}{\partial M_t} = -\frac{1}{\frac{1}{1 + \beta} \frac{Y_t - T_t}{Y_t} \frac{A_t}{(M_t - A_t)^2}} < 0 \tag{10.58}$$
An increase of the current money supply reduces the nominal interest rate as it increases liquidity in the economy. The effect is similar to the liquidity effect, which we discussed in Section 10.3, analyzing the short-term effects of an increase in the money supply.

10.7.4 The Liquidity Effect in Representative Household Models

The liquidity effect is not so obvious in representative household models. It occurs only as a result of temporary increases in the money supply.

This is illustrated by examining the equations for determining the nominal interest rate, i.e (10.53) for the model with money in the utility function, and (10.37) for the model with the cash advance constraint.

Let us look at the latter. The nominal interest rate is determined by,

\[
\frac{1}{1 + i_{t+1}} = \beta E_t \left( \frac{M_t}{M_{t+1}} \right) \quad (10.37)
\]

If there is a temporary increase of the current money supply, which does not affect the expectation of the future money supply, then the impact on the nominal interest rate is given by,

\[
\frac{\partial i_{t+1}}{\partial M_t} = -\frac{1}{M_t E_t \left( \frac{M_t}{M_{t+1}} \right)} < 0 \quad (10.59)
\]

From (10.59) it is demonstrated that there is liquidity result for temporary increases in the money supply. Similar properties apply to the model with money in the utility function of a representative household (Equation 10.53).

In contrast, if there is a permanent increase in the money supply, which does not affect the expected ratio between \(M_t\) and \(M_{t+1}\), then there is no impact on the nominal interest rate. There is no liquidity effect for permanent increases in the money supply.

Finally, if there is an increase in the money supply which increases the expected ratio between \(M_{t+1}\) and \(M_t\), then not only is there no liquidity effect, but there is the opposite, i.e. a positive impact on the nominal interest rate by an increase in the money supply.

For example, assume that the growth rate of the money supply follows a linear first order, stationary autoregressive stochastic process of the form,

\[
\ln \left( \frac{M_t}{M_{t-1}} \right) = \mu (1 - \lambda) + \lambda \ln \left( \frac{M_{t-1}}{M_{t-2}} \right) + \epsilon_t \quad (10.60)
\]

where \(0 < \lambda < 1\) and \(\epsilon_t\) is a white noise process.
It is simple to prove that a positive disturbance $\varepsilon_t$ leads to higher nominal interest rates, as it causes a temporary increase in the expected ratio between $M_{t+1}$ and $M_t$. Consequently, a positive, albeit temporary, positive disturbance in the growth of the money supply leads to higher nominal interest rates. The reason is that it increases the expected future increase in the money supply and thus expected future inflation. This effect is often referred to in literature as the \textit{liquidity puzzle}.

The liquidity puzzle is not the only paradoxical property of money demand models of a representative household. A second paradox is the indeterminacy of the price level when the central bank pegs the nominal interest rates instead of the money supply. This has led to large literature on interest rate pegging and the development of the so-called \textit{fiscal theory of the price level}.

10.7.5 Interest Rate Pegging and Price Level Indeterminacy

We shall next analyze interest rate pegging in the representative household models with money in the utility function and cash-in-advance constraints. Whereas under a money supply rule, such as the ones we have analyzed so far, the price level and its rate of change are uniquely determined in such models, if the central bank pegs the interest rate, then the price level and the money supply cannot be determined uniquely. This result is known as \textit{price level indeterminacy}, and was first alluded to by Wicksell (1898) in the context of a static traditional ad hoc monetary model, and, more recently, by Sargent and Wallace (1975), in the context of an ad hoc macro model with rational expectations.

For simplicity, we will assume that there is no uncertainty, and that the nominal interest rate is pegged by the central bank at a constant level $i_0$.

From the money demand function of the representative household model with money in the utility function, the money demand equation implies that,

$$
\frac{M_t}{P_t} = \left( \frac{\gamma - i_0}{1 - \gamma (1 + i_0)} \right)^{1-x} Y_t
$$

In (10.61) we have imposed the equilibrium condition in the goods market, that $C_t = Y_t$. Given that real income $Y_t$ is exogenous, this condition is satisfied for an infinite number of combinations of $M$ and $P$. For given $Y$, if it is satisfied for $M_0$ and $P_0$, it is also satisfied for $\lambda M_0$ and $\lambda P_0$, for any $\lambda$. Thus, both the money supply and the price level are \textit{indeterminate}.

The reason for the indeterminacy is that, under interest rate pegging, there is no monetary anchor which can determine the price level, as in the case where the central bank determines the money supply. Since the central bank is committed to providing unlimited credit at a nominal interest rate $i_0$, then the money supply is determined by the demand for money. Neither the price level, nor the money supply can be identified uniquely. The equilibrium condition for money demand can be satisfied with both high prices and a consequent high stock of money, and with low prices and a consequent low stock of money, i.e. virtually for any level of prices.$^8$

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$^8$ As Sargent and Wallace (1975) were the first to recognize, this indeterminacy was first alluded to by Wicksell (1898), in the context of a static monetary analysis with flexible prices.
The same problem arises in the cash-in-advance representative household model. From the Euler equation for consumption (10.35), and the goods market equilibrium condition \( C_t = Y_t \) for every \( t \), it follows that,

\[
P_{t+1}Y_{t+1} = \beta(1 + i_0)P_tY_t \tag{10.62}
\]

As income is exogenous, and the nominal interest rate fixed, (10.62) is satisfied for any price level. Multiplying \( P_t \) and \( P_{t+1} \) by a coefficient \( \lambda \), (10.62) continues to be satisfied, because it linearly homogeneous in prices. The price level is thus indeterminate.

Again, the reason for the indeterminacy is that, under interest rate pegging, there is no monetary anchor which can determine the price level, as in the case where the central bank determines the money supply. Neither the price level, nor the money supply can be identified uniquely under interest rate pegging. The equilibrium condition for consumption can be satisfied with both high prices and a consequent high stock of money, and with low prices and a consequent low stock of money, i.e. virtually for any level of prices.

This indeterminacy is especially problematic as the key monetary tool for most central banks is not the money supply, but nominal interest rates. How is it possible to determine the price level in this case?

### 10.7.5 Solutions to the Price Level Indeterminacy Problem under Interest Rate Pegging

One of the first answers to this problem had been provided by the monetary economist who first realized its existence, namely Wicksell (1898). Wicksell proposed that, “So long as prices remain unaltered, the banks’ rate of interest is to remain unaltered. If prices rise, the rate of interest is to be raised; and if prices fall, the rate of interest is to be lowered; and the rate of interest is henceforth to be maintained at its new level until a further movement of prices calls for a further change in one direction or the other.” (p. 189).

Wicksell’s rule can be written as,

\[
i_t = \rho + \phi\frac{P_t - P_0}{P_0} \tag{10.63}
\]

where \( P_0 \) is the target price level of the central bank.

Substituting (10.63) for \( i_0 \) in (10.62), assuming that output is constant, and taking logs, we get,

\[
\ln P_t = \frac{1}{1+\phi} \ln P_{t+1} + \frac{\phi}{1+\phi} \ln P_0 \tag{10.64}
\]

If \( \phi \) is positive, as Wicksell proposed, then (10.64) is a stable difference equation, which fully determines the price level. The price level is uniquely defined, as it adjusts immediately to \( P_0 \), the
target price level of the central bank. If $\phi$ is equal to zero, then we have price level indeterminacy, as in equation (10.62).\footnote{The same analysis can be carried out in the context of equation (10.61), the money market equilibrium condition of the representative household model with money in the utility function.}

Wicksell’s rule is a good example of a stabilizing interest rate rule, which makes the nominal interest rate not exogenously determined by the central bank, but a function of endogenous variables, such as the price level, about which the central bank is concerned.

Alternative ways to solve the problem of price level indeterminacy when the policy instrument of the central bank is the nominal interest rate, have been proposed since. Inflation targeting rules, nominal income rules, and more recently the Taylor (1993) rule, which is a generalization of Wicksell’s rule. We shall examine the properties of such rules in the chapters on aggregate fluctuations (Chapter 13) and monetary policy (Chapter 16).

One theoretical development worth mentioning in this context is the so-called fiscal theory of the price level (see. Leeper 1991, Sims 1994 and Woodford 1994, 1995). This theory argues that even if monetary policy is not sufficient to determine the price level, the price level can be determined at the level which ensures that public debt, which is defined in nominal terms, does not follow an explosive path. A path for the price level that ensures a path of nominal public debt that satisfies the inter-temporal budget constraint of the government is sufficient in those models to determine the price level.

It is finally worth stressing that the problem of price level indeterminacy under interest rate pegging does not arise in overlapping generations models. Unlike the representative household model, where both the current and the future price level are non predetermined variables, in the overlapping generations model, the price level is determined through the predetermined nominal financial assets of “old” households. These function as a monetary anchor and help in determining the price level.

For example, in the cash in advance version of the Samuelson overlapping generations model, the equilibrium condition in the goods and services market is given by (10.48). Assuming that the nominal interest rate is pegged at $i_0$ by the central bank, one can solve (10.48) for the price level as,

$$P = \frac{(1 + \beta)(1 + i_0)A_t}{(\beta + (1 + \beta)i_0)Y_t + T_t}$$

(10.65)

The price level is uniquely defined. Since consumption depends on the real value of financial assets of the old households, and these assets are positive, the price level is determined, regardless of the interest rate policy of the central bank.

In traditional ad hoc monetary models, the dependence of consumption on the financial wealth of households was called the Pigou effect (see. Pigou 1943), or the real balance effect (see Patinkin 1956). As Sargent and Wallace (1975) had indicated in their original analysis of price level
indeterminacy, in the presence of a Pigou or real balance effect, the problem does not arise even if 
the central bank pegs the nominal interest rate.\textsuperscript{10}

\textbf{10.8 Seigniorage and Inflation}

If the growth of the money supply translates into higher inflation in the longer term, why don’t 
governments and central banks keep the rate of growth of the money supply low and stable in order 
to control and eliminate inflation?

The answer is that governments often have other policy motives besides the motive of tackling 
inflation. Perhaps the most important incentive for the issuance of new money by governments is to 
finance expenditure that they cannot, or do not want to, finance through other methods, such as 
higher taxes or higher government debt.

The main cause of all the episodes of \textit{high inflation} or \textit{hyperinflation} appears to have been the need 
of governments to use revenue from money creation (seigniorage) to finance wars and war 
reparations, revolutions, extraordinary costs related to natural disasters or sudden reductions in their 
borrowing capacity from financial markets and their capacity to raise revenue from taxes and 
customs revenues.

In this section we explore the relationship between the growth rate of money supply, inflation and 
the needs of governments to raise revenue through seigniorage. We examine both the case in which 
the required income from seigniorage can be raised on the balanced growth path, a situation in 
which equilibrium inflation turns out to be high, and the situation in which the required revenue 
from seigniorage is so high, that it cannot be raised in steady state equilibrium, which can lead to 
hyperinflation.

The generally accepted definition of hyperinflation is due to Cagan (1956). Cagan defined a a 
period of hyperinflation as one “beginning in the month in which the rise in prices exceeds 50% and 
as ending in the month before the monthly rise in prices drops below that amount and stays below 
for at least a year.”

The first modern periods of hyperinflation occurred in Europe in the aftermath of World War I, as 
well as during and in the aftermath of World War II.

In the last forty years very high inflation and hyperinflation reappeared in some Latin American 
countries, in some transition economies after the collapse of the Soviet Union and in some 
belligerent countries of Asia and Africa. Moreover, many countries, without reaching the levels of 
hyperinflation, have experiences with high inflation from 100\% to 1000\% per year for quite long 
periods.\textsuperscript{11}

\textsuperscript{10} “In both our model and the standard static model, the aggregate demand schedule must exclude any components of 
real wealth that vary with the price level if Wicksell’s indeterminacy is to arise. For example, if the anticipated rate of 
capital gains on real (outside) money balances is included in the aggregate demand schedule, the price level is 
determinate with a pegged interest rate.” (Sargent and Wallace 1975, footnote 5, page 251). This is exactly what 
happens in the cash in advance overlapping generations model, and this is the reason that the problem of price level 
determinacy does not arise in the context of this model.

\textsuperscript{11} Apart from the study of Cagan (1956), for the hyperinflations of the interwar period and the Second World War, see 
Sargent (1982) on how four hyperinflations ended. More recent episodes of high inflation and hyperinflation have been 
10.8.1 Monetary Growth, Inflation and Seigniorage

In order to study the relation between the rate of growth of the money supply, inflation and revenue from seigniorage, we shall start from the general money demand function (10.2). In equilibrium, the demand for money equals the supply of money, so it follows that,

$$\frac{M}{P} = m(Y, i)$$  \hspace{1cm} (10.66)

In order to simplify matters, we shall use a linear logarithmic form of the money demand function $m$.

$$\frac{M}{P} = \kappa Y e^{-\eta i}$$  \hspace{1cm} (10.67)

where $\kappa$ is a constant, $e$ the basis of natural logarithms, and $\eta > 0$ the semi-elasticity of money demand with respect to the nominal interest rate $i$.

The nominal interest rate is defined by the Fisher equation,

$$i = r + \pi^e$$  \hspace{1cm} (10.68)

where $r$ is the real interest rate and $\pi^e$ is expected inflation.

Real output $Y$ is considered exogenous, and it is assumed that it grows at a rate $g+n>0$, while the rate of growth of the nominal money supply $M$ is equal to $\mu>0$.

Under these assumptions, inflation on the balanced growth path is determined by,

$$\pi = \mu - (g + n)$$  \hspace{1cm} (10.69)

Assuming rational expectations, we can substitute (10.69) in (10.68), and the resulting equation in (10.67). The money demand function can thus be written as,

$$\frac{M}{P} = \kappa Y e^{-\eta (r + \mu - (g+n))}$$  \hspace{1cm} (10.70)

To further simplify matters, we shall assume that the golden rule applies on the balanced growth path, which implies that $r=g+n$. Under this additional assumption, (10.70) simplifies to,

$$\frac{M}{P} = \kappa Y e^{-\eta \mu}$$  \hspace{1cm} (10.71)
Because of the golden rule, the nominal interest rate is equal to the rate of growth of the money supply $\mu$.\(^{12}\)

We can now define seigniorage revenue $S$. This is equal to the real resources that the government commands by issuing additional money, and buying goods and services. Seigniorage is thus given by,

$$S = \frac{\dot{M}}{P} = \frac{M \cdot \dot{P}}{M \cdot P} = \mu \frac{M \cdot P}{P} = \mu \kappa Y e^{-\eta \mu} \quad (10.72)$$

where $S$ denotes total seigniorage revenue from money creation. As a proportion of total output, seigniorage revenue is defined by,

$$s = \frac{S}{Y} = \mu \frac{M}{PY} = \mu \kappa e^{-\eta \mu} \quad (10.73)$$

where $s$ is seigniorage revenue relative to total output.

Taking the first derivative of (10.73) with respect to $\mu$, we can see how seigniorage revenue with respect to output depends on the rate of growth of the money supply.

$$\frac{\partial s}{\partial \mu} = (1 - \eta \mu) \kappa e^{-\eta \mu} \quad (10.74)$$

(10.74) is positive for as long as the rate of growth of the money supply $\mu$ is smaller than $1/\eta$. When $\mu$ exceeds $1/\eta$, the change in seigniorage revenue as a proportion of total output when $\mu$ increases further becomes negative. For $\mu > 1/\eta$ a further increase in the rate of growth of the money supply has a negative effect on government revenue from money creation. This happens because the reduction in real money holdings by household and firms, which is the basis of this revenue, is greater than the rise in $\mu$.

10.8.2 The Seigniorage Laffer Curve

The revenue from seigniorage as a percentage of total output, as a function of the growth rate of the money supply are depicted in Figure 10.8. As can be seen from Figure 10.8, the revenue from money creation is characterized by a Laffer curve, because up to a point the rise in the growth rate of the money supply increases revenues from seigniorage as a percentage of output, but after a point it begins to reduce them, because the reduction in real money demand exceeds the rise in the rate of growth of the money supply.\(^{13}\)

\(^{12}\) Alternatively, we could assume that the real interest rate equals $\rho + g$, as would apply on the balanced growth path of a representative household model. In this case, the nominal interest rate would be equal to $\rho - n + \mu$. The results of the analysis would be similar, as in periods of high inflation and hyperinflation the growth rate of the money supply is much higher than the pure rate of time preference $\rho$ and the population growth rate $n$.

\(^{13}\) The term Laffer Curve derives from Arthur Laffer, an economist who claimed, in a meeting with administration officials in the USA in 1974, that tax revenue after a point becomes a negative function of tax rates, because of the disincentive effects of high taxes. He famously sketched this curve on a napkin in the revenue where the meeting took place. Laffer himself notes antecedents in the writings of the 14th-century social philosopher Khaldun and Keynes.
It is interesting to calculate at what percentage of total output is seigniorage revenue maximized. The maximum seigniorage revenue $s_{\text{max}}$, that can be extracted occurs when $\mu = 1/\eta$. From (10.73) it follows that,

$$s_{\text{max}} = \frac{\kappa}{\eta \epsilon}$$  \hspace{2cm} (10.73')

Cagan, using annual data, estimated that $\eta$ lies between 1/2 and 1/3. Consequently, he estimated the growth rate of the money supply that maximizes revenues from seigniorage, as a percentage of total output, and the corresponding inflation, at between 200% and 300% per year. Assuming that $\kappa = 0.10$ in (10.71), the maximum revenue from seigniorage as a percentage of total output is between 7-11%. This is roughly the estimate of Cagan (1956). For the period 1975-1985, Sachs and Larrain (1993) estimated actual revenue from seigniorage at about 5 to 6.5% for high inflation countries such as Italy, Bolivia, Turkey, and Peru, and much lower for a series of other countries.

10.8.3 A High Inflation Equilibrium and the Transition to Hyperinflation

Let us now consider a government which needs to fund a proportion of its public spending through seigniorage. We will assume that this financing requirement, as a proportion of total output is equal to $S_E$, which is less than the maximum seigniorage $s_{\text{max}}$ that the government can achieve by setting the growth rate of the money supply at $\mu = 1/\eta$. The equilibrium is depicted shown in Figure 10.10. There are two options to achieve revenue equal to $S_E$. One is with a growth rate of the money supply $\mu < 1/\eta$, and the other is with a growth rate of the money supply $\mu > 1/\eta$. We will assume that the government dislikes inflation, and therefore chooses the lowest growth rate of the money supply that is compatible with the objective of raising revenue $S_E$ from seigniorage. For as long as the government needs to finance a proportion $S_E$ of its output through seigniorage, the economy is trapped in an equilibrium with a rate of growth of the money supply equal to $\mu_E$ and the corresponding high inflation. For example, if the government wants to raise seigniorage corresponding to 6% of total output, assuming $\eta = 1/2$, this implies an annual growth rate of the money supply (and corresponding steady state inflation) equal to about 100%.

But how can a hyperinflation arise? Unlike a high inflation, a hyperinflation is a disequilibrium phenomenon. It arises when a government tries to raise seigniorage that exceeds the maximum seigniorage that can be raised in the steady state.

Now suppose a government which needs to raise seigniorage which, as a proportion of total output, is higher than the maximum that can be raised in the steady state. We assume $S_E > S_{\text{max}}$. Obviously there can be no balanced growth path in which the government can raise revenues from seigniorage to exceeds $S_{\text{MAX}}$. However, for a time, and as the economy adjusts towards the balanced growth path, the government may be able to raise seigniorage revenues greater than $s_{\text{max}}$. This could happen if, for example, there is gradual adjustment in the demand for money, or gradual adjustment in inflationary expectations.

Suppose that the demand for money does not adjust immediately to its steady state level after a change in the nominal interest rate, but only adjusts gradually. Thus, when the nominal interest rate increases, money demand is temporarily higher than in the steady state. In this case, during the adjustment, the monetary base upon which the inflationary tax is imposed is higher than the steady
state monetary base. Consequently, during the adjustment, as $\mu$ increases, seigniorage revenues will exceed $s_{max}$ because real money balances are higher than on the balanced growth path. As the demand for money decreases gradually, the government should constantly increase the rate of monetary expansion and the consequent inflation, to be able to have the required high revenues from seigniorage. This can lead to explosive path for the rate of growth of the money supply and a consequent hyperinflation.

Let us then assume that (10.71) defines the “steady state” money demand function, and that actual money demand adapts to its steady state level only gradually. We shall continue to assume that real output and the real interest rate are on their exogenous balanced growth paths. From (10.71), the steady state demand money demand as a percentage of total output $m^*$, depends negatively on the growth rate of the money supply, and given by,

$$m^* = \frac{M}{PY} = \kappa e^{-\eta \mu} \quad (10.75)$$

In the short run, real money demand adjusts gradually towards its steady state value according to,

$$\frac{d \ln m(t)}{dt} = \frac{\dot{m}(t)}{m(t)} = \psi \left( \ln m^* - \ln m(t) \right) \quad (10.76)$$

where $0 < \psi < 1/\eta$.

The concept behind the assumed gradual adjustment is that it is difficult for economic agents to adapt their habits regarding the financing of their transactions, or use alternative means of payment in the short run, and, that, hence, the demand for real money balances adjusts gradually over time. The particular log-linear functional form is chosen for convenience. The parameter $\psi$, which measures the speed of adjustment, is assumed to be positive but less than $1/\eta$. We thus assume that adjustment is not too fast.

Substituting (10.75) in (10.76),

$$\frac{\dot{m}(t)}{m(t)} = \psi \left( \ln \kappa - \eta \mu(t) - \ln m(t) \right) \quad (10.77)$$

(10.77) describes the evolution of the short run demand for money. It is worth noting that in (10.77) we have allowed the rate of growth of the money supply $\mu$ to depend on time.

Let us now turn to the government’s financing needs. We assume that the government needs revenues from seigniorage which may exceed the maximum that can be raised on the balanced growth path, i.e that $s_E > s_{max}$. Although such revenues cannot be raised on the balanced growth path, they can be raised along the adjustment path, i.e before money demand has fully adjusted to its balanced growth path level. We shall show that if indeed $s_E > s_{max}$, then this requires an ever-increasing rate of growth in the money supply and ever-increasing inflation.

For the government to achieve its target $s_E$, the following relation must hold continuously,
The (10.79) suggests that in order to maintain revenues from seigniorage constant as a percentage of total income at the level $s_E$, the growth rate of the money supply must keep increasing continuously, at the same rate as the decline of real money demand relative to output.

Substituting (10.79) and (10.78) in (10.77) we get,

$$
\frac{\mu'}{\mu} = -\frac{m}{m'} \tag{10.80}
$$

From (10.80), for the rate of growth of the money supply to be stabilized, a necessary and sufficient condition is that,

$$
s_E = \mu e^{-\eta} \leq s_{\text{max}} \tag{10.81}
$$

If $s_E > s_{\text{max}}$, then the right hand side of (10.80) is positive for all rates of growth of the money supply. If we take the first derivative of (10.80) with respect to $\mu(t)$, we shall realize that after a point, a higher rate of growth of the money supply leads to a higher rate of change of the growth of the money supply, with the result an explosive path for the rate of growth of the money supply and inflation.

The relationship between the percentage change in the growth rate of the money supply and the rate of change in the money supply provided from (10.78), for different financing requirements from seigniorage, is depicted in Figure 10.10.

In the case where the financing needs of the government from seigniorage are less than or equal to the maximum possible on the balanced growth path, then the rate of growth of the money supply stabilizes at a rate that may indeed entail significant inflation, but inflation is stable and does not evolve into hyperinflation.

However, if the financing needs of government exceed the maximum that is sustainable on the balanced growth path, then, as the government tries to raise the necessary revenue from seigniorage, the rate of growth of the money supply gradually accelerates, in order to keep up with the declining monetary base, and the economy falls into a state of hyperinflation. The reason is that inflation
gradually reduces the demand for money relative to total output, and the government needs an ever increasing growth rate of the money supply in order to be able to collect the needed seigniorage revenue.

Our analysis of the link between the needs of a government to raise seigniorage in order to finance government expenditure, and the rate of growth of the money supply, can thus help explain episodes of high inflation, or even hyperinflation.

Our basic analysis explains why, in many cases, inflation may be driven to very high levels. This is due to the inability of a government to finance its spending from other revenue sources, such as taxation or borrowing from the markets, and its need to use seigniorage from money creation.

The analysis also explains why even though inflation may reach very high levels, it is not necessary that it will evolve into an explosive hyperinflation. For this to happen, the financing needs of the government must be so high that they exceed the maximum level that can be financed with high inflation on a balanced growth path.

Finally, the analysis emphasizes the central role of fiscal problems as the main root causes of both high inflation and hyperinflation. A significant precondition for tackling high inflation or hyperinflation is to pursue reforms that address the underlying fiscal problems (Sargent 1982).

### 10.9 Conclusions

In this chapter we have analyzed the role and functions of money. Money performs three functions. First, it is a unit of account, second, it is a generally accepted means of payment, and, thirdly, it is a store of wealth.

We first reviewed the basic functions of money and the factors that determine the demand for and supply of money. We analyzed the concept of short run equilibrium in the money market, assuming that the central bank follows a policy of either targeting the money supply or pegging nominal interest rates, and also defined the notion of the long-term neutrality of money.

We then focused on a number of dynamic general economic equilibrium models with money, in order to analyze the determination of the price level and nominal interest rates and also analyzed the long relationship between the money supply, the price level and inflation.

Finally we examined the fiscal incentive for increasing the money supply and its effects on inflation. The most important motive for sustained large increases in the money supply by governments has been the incentive to finance government expenditure that could not be financed by other methods, such as additional taxes or government bonds. This source of revenue for the government is called seigniorage. The main cause of all episodes of sustained high inflation or even hyperinflation, has been the need of governments to use their privilege of printing money, in order to obtain seigniorage.

We investigated the relationship between the growth rate of the money supply, inflation and government revenue from seigniorage. We examined both the situation where the revenues from seigniorage are adequate for the needs of a government on the balanced growth path, a situation in
which inflation turns out to be high but stable, and the situation in which the required seigniorage revenues are not sufficient, which may lead to hyperinflation.
References


Figure 10.1
The Demand for Money and the Nominal Interest Rate

$m(Y)$

Nominal Interest Rate, $i$

Demand for Real Money Balances, $M^d/P$
Figure 10.2
An Increase in Real Income and the Demand for Money
Figure 10.3
Short Run Equilibrium in the Money Market
Figure 10.4
Short Run Effects of a Rise in the Money Supply: The Liquidity Effect

![Graph showing the relationship between nominal interest rate and real money balances. The graph illustrates the liquidity effect where an increase in the money supply leads to a decrease in the nominal interest rate at a given level of real money balances.](image-url)
Figure 10.5
Short Run Effects of a Rise in Real Income
Figure 10.6
Equilibrium in the Money Market with Interest Rate Pegging
Figure 10.7
Money Demand Indeterminacy in the Samuelson Overlapping Generations Model
Figure 10.8
The Relationship between Seigniorage Revenue and the Rate of Growth of the Money Supply

\[ s(\mu) = \mu \theta e^{\gamma \mu} \]

\[ s_{\text{max}} \]

\[ \mu = 1/\eta \]
Figure 10.9
Equilibrium with High Inflation
Figure 10.10
Equilibria with High Inflation and Hyperinflation

\[ \frac{\dot{\mu}}{\mu} \]

- \( S_E > S_{max} \)
- \( S_E = S_{max} \)
- \( S_E < S_{max} \)

\( \mu_E \), \( \mu = \frac{1}{\eta} \), \( \mu_{E'} \)