Chapter 16
Unemployment and Matching in the Labor Market

In a fully competitive labor market, without uncertainty and frictions, employers would be indifferent about whether an employee leaves her job, since they can replace her immediately and at no cost, and at the same competitive wage, with another employee. Accordingly, an employee would be indifferent about losing her job, since she can readily find another one at the same competitive real wage. Moreover, in such a market “involuntary” unemployment cannot exist, because the excess supply of workers would cause an immediate decline in real wages, which would lead to the elimination of unemployment.

In almost all economies there is a positive and non-trivial unemployment rate even in boom periods. There are many unemployed people seeking jobs similar to those held by workers with similar characteristics, at wages equivalent to those generally prevailing in the labor market. At the same time, there are many firms with vacancies, seeking to fill them with employees, possessing characteristics similar to those of the unemployed, and at prevailing real wages. How can then one explain the existence and the fluctuations of both unemployment and vacancies?

The explanation of unemployment is one of the central tasks of macroeconomics. There are two types of questions that are being asked. First, what determines the equilibrium rate of unemployment in an economy, that is the “natural” rate, what are its implications, and to what extent does the equilibrium unemployment rate reflects labor market distortions. The second key question concerns the fluctuations of the unemployment rate during the economic cycle.¹

Cyclical fluctuations in unemployment can be explained by and large by new keynesian models with labor market distortions and nominal wage and price contracts, as the one we have put forward in the previous chapter. In this chapter we shall delve deeper into the microeconomic foundations of models of the determinants of the equilibrium unemployment rate, when there are real labor market frictions. We shall focus on one of the most important models of this latter category, the Mortensen-Pissarides model.

16.1 Alternative Views of the Labor Market and Equilibrium Unemployment

There are many alternative approaches to the modeling of the labor market that differ from the competitive model of the labor market. All these approaches offer an alternative explanation as to why, despite unemployment and vacancies, real wages do not adjust in order to absorb the unemployed and eliminate unemployment.

One approach consists of the so called *efficiency wage* theories. In these theories there is asymmetric information. Firms cannot observe either the productivity or the effort of workers directly. Thus, firms offer wages above the average productivity of job seekers, or existing employees, in order, either to attract workers with above average productivity, or to provide incentives to their employees to work more intensively. They are therefore not prepared to reduce real wages, or to replace workers already in jobs with the unemployed, even if the unemployed offered to work at lower wages.\(^2\)

A second class of theories are theories of long-term contracts. These contracts prevent firms from undertaking unilateral changes to wages in response to shocks, if this is not provided for in the long-term contract. The contracts can be *explicit*, such as collective, industry and individual employment contracts or informal and *implicit*.\(^3\)

Finally, there are theories that highlight the search costs of looking for an appropriate job, by unemployed job seekers, and for an appropriate employee, by firms with vacancies. In these theories, a costly search process is required for the matching of job seekers with appropriate vacancies in order to create a new job. These theories are called *search or matching* theories of the labor market.

The basic idea of search or matching models of the labor market is that the labor market functions in a decentralized and uncoordinated way, and that job creation is a costly process that requires time for both job seekers and firms. Consequently, jobs entail rents, something that does not apply in fully competitive labor market models.

In the remainder of this chapter we shall focus on one of the most important models of this latter category, the Mortensen-Pissarides model.\(^4\)

### 16.2 The Matching Function

A key assumption of this class of models is that the number of jobs created at each moment is a positive function of the number of firms looking for employees and the number of unemployed job seekers. The outcome of this process is described by the so-called *matching function*.

The number of jobs created at each moment in time is given by,

\[
 mL = m(uL,vL) \tag{16.1}
\]

where,

---

\(^2\) See Weiss (1980) and Shapiro and Stiglitz (1984) for the two most important early models based on this approach.

\(^3\) For the original implicit contract theories see Azariadis (1975), Baily (1974) and Gordon (1974). These theories viewed employment contracts as insurance contracts between risk neutral employers and risk averse employees, against adverse shocks to employment. For theories that are based on explicit negotiations see McDonald and Solow (1981). For theories that distinguish between insiders and outsiders in the labor market see Lindbeck and Snower (1986), Blanchard and Summers (1986), Gregory (1986) and Gottfries (1992). The model we used in Chapter 14, where the nominal wage is determined by periodic nominal contracts between firms and labor market insiders, is a simple stylized model of this latter category.

The matching function is assumed to be increasing in every one of its arguments, concave and linearly homogeneous. Thus, it is characterized by constant returns to scale. These are the same assumptions as the ones for the neoclassical production function. The higher the number of unemployed job seekers, and the higher the number of vacancies, the higher the number of jobs that are created. If you double the number of unemployed and vacancies, the number of jobs created will also double.

Assuming that all vacancies have the same probability of being filled, and that all the unemployed have the same probability of being employed, the probability of filling a vacancy shall be equal to the ratio of the number of new jobs created over all existing vacancies. If we define this probability as \( q \), then the probability of filling a vacancy is defined by,

\[
q = \frac{m(uL,vL)}{vL} = \frac{m}{\frac{u}{v}L} = \frac{m}{\frac{u}{v}L,1}
\]

(16.2)

We have assumed that every unemployed job seeker has the same probability of finding a job, and that every vacancy has the same probability of being filled. We have thus assumed a Poisson frequency distribution.

From (16.2), the probability of filling a vacancy is a function only of the ratio of the unemployed to job vacancies. This is due to the assumption that the matching function is linearly homogeneous. The more the unemployed per vacancy, the greater will be the likelihood of filling any particular vacancy.

We define as \( \theta \), the ratio of vacancies to the unemployed.

\[
\theta = \frac{v}{u}
\]

(16.3)

\( \theta \) measures the degree of tightness in the labor market. The higher the number of vacancies relative to the unemployed, the greater the tightness of the labor market.

We can express (16.2) as a function of labor market tightness,

\[
q = q(\theta)
\]

(16.4)

From the properties of the matching function, it follows that,

\[
q'(\theta) \leq 0, \quad -1 < \eta(\theta) = \frac{\theta q'(\theta)}{q(\theta)} < 0
\]
where $\eta(\theta)$ is the elasticity of $q$ with respect to $\theta$. Higher tightness in the labor market implies a lower probability of filling a vacancy.

It is also worth noting, that, because of the assumption of a Poisson density function, the average expected duration of a vacancy is equal to the inverse of $q$, and is thus a positive function of $\theta$.

The probability of finding a job, by an unemployed job seeker, is correspondingly equal to,

$$\frac{m(uL,vL)}{uL} = \frac{v}{u} \cdot \frac{m(uL,vL)}{vL} = \theta q(\theta)$$

From (16.5), the elasticity of the probability of finding a job with respect to $\theta$, is given by,

$$1 - \eta(\theta) > 0$$

Therefore, the higher the tightness in the labor market (a higher $\theta$), the higher the probability of an unemployed job seeker to find a job.

The average expected duration of unemployment is given by $1/\theta q(\theta)$ and is a negative function of $\theta$.

It is worth noting that, in models of this type, the price mechanism cannot lead the probability of filling a vacancy or the probability of finding a job to unity, as the labor market does not function only via the price mechanism, but also via the degree of tightness of the labor market, which determines the probabilities of firms to fill their vacancies or of the unemployed to find jobs in any particular instance.

The dependence of the probability of filling a vacancy and the probability of finding a job on the relative number of vacancies to the unemployed (tightness) is an example of a trading externality. These search externalities are important for the properties of equilibrium unemployment in these models.

### 16.3 Flows Into and Out of Employment and Equilibrium Unemployment

The model assumes that a proportion of existing jobs are terminated in every instant. The destruction of jobs and the flow from employment to unemployment, is due to either cyclical or structural real disturbances that make them unprofitable. It is assumed that at any instant the probability of destruction of any job is equal to $\lambda$, where $\lambda$ is an exogenous parameter.\(^5\)

On the other hand, job creation occurs when a firm and an employee agree to sign a contract with a wage which is the result of a bilateral negotiation. This leads to a flow out of unemployment.

Therefore, at any given moment there are two flows in the labor market. One flow is from existing jobs into unemployment, because of job terminations, and the other from unemployment to newly created jobs. The change of the unemployment rate is thus described by,

\(^5\) Mortensen and Pissarides (1994) have generalized this model to make the job termination rate endogenous. This does not fundamentally change the properties of the model. In the interests of simplicity we shall thus stick to the model with an exogenous job termination rate $\lambda$. 
The change in the unemployment rate at each point in time depends on the difference in the proportion of jobs destroyed from the proportion of new jobs created, the proportions being defined relative to the labor force.

In the steady state, the unemployment rate will be constant. Consequently, the equilibrium unemployment rate is determined by the condition,

\[ \lambda(1-u) = \theta q(\theta)u \] \hspace{1cm} (16.7)

which implies,

\[ u = \frac{\lambda}{\lambda + \theta q(\theta)} \] \hspace{1cm} (16.8)

(16.8) is the first key equation of this model. The equilibrium unemployment rate depends positively on \( \lambda \), the exogenous rate of termination of a job, and negatively on labor market tightness \( \theta \). Labor market tightness is an endogenous variable in this model, and is determined in the labor market.

The negative relationship that (16.8) implies between the unemployment rate and labor market tightness \( \theta \), or, equivalently, between the unemployment rate and the vacancy rate, is usually called the Beveridge curve, and is depicted in Figure 16.1. The Beveridge curve defines just a negative relation between vacancies and unemployment. In order for unemployment to be determined, one needs to know labor market tightness, which is one of the endogenous variables in this model.

Assume that labor market tightness is equal to \( \theta_E \). Then the steady state unemployment rate would be determined at \( u_E \), as in Figure 5.1, since for given labor market tightness, equilibrium unemployment is determined on the Beveridge curve.\(^6\)

We next turn to the endogenous determination of labor market tightness.

**16.4 Firms and the Creation of Vacancies and Jobs**

We have the creation of a new job when a prospective employer and a prospective employee get together and agree to an employment contract. Of course, before this happens, the potential employer has to create a vacancy and search for an employee and the prospective employee has to be unemployed and looking for a new job. All this involves time and costs and is described by the matching function.

We assume that the instantaneous value of the product of a job is constant and equal to \( p > 0 \). The instantaneous cost of a vacant post to a prospective employer is equal to \( pc \) where \( 0 < c < 1 \). During the period of search, the employer faces a probability \( q(\theta) \) of finding a a suitable employee, which is independent of her actions.

\(^6\) See Beveridge (1944) who first empirically identified this relationship.
The number of vacancies is endogenous and is determined by profit maximization. Any firm can create a vacancy and search for employees. Profit maximization with free entry means that the expected profit from the marginal vacancy should be equal to zero. Otherwise, firms will continue to create vacancies.

16.4.1 The Present Value of Net Expected Profits from an Existing Job

We denote by $J$ the present value of net expected profits from an existing job.

The instantaneous gross profit from a job for a firm is equal to the difference $p-w$, where $w$ is the real wage. However, in every instant, there is an exogenous constant probability of the job ending, and the firm losing the present value of net expected profits from the job.

Thus, the instantaneous expected net profit from a job is equal to,

$$p - w - \lambda J$$ 

(16.9)

Assuming a perfect capital market and an infinite horizon, the present value of this expected net profit is equal to,

$$J = \int_0^\infty e^{-rt} (p - w - \lambda J) dt = \frac{p - w - \lambda J}{r}$$ 

(16.10)

$r$ is the real interest rate which is assumed exogenous and constant.\(^7\)

From (16.10), the present value $J$ of a job for the employer must satisfy the Bellman equation,

$$rJ = p - w - \lambda J$$ 

(16.11)

Thus, from (16.11), the instantaneous net opportunity cost of a job $rJ$ must be equal to the instantaneous net expected profit from the job. This implies the condition,

$$w = p - (r + \lambda)J$$ 

(16.12)

For a positive net present value of expected profits from an existing job, the real wage will be less than productivity, both because of the interest costs of maintaining a job, and the positive probability of termination of the job. Thus, the wedge between productivity and the real wage must reflect these capital and insurance costs.

16.4.2 The Present Value of Net Expected Profits from a Vacancy and the Creation of Vacancies

We denote by $V$ the present value of expected profits from a job vacancy.

---

\(^7\) See Pissarides (2000) on how the real interest rate can become endogenous.
The *instantaneous net expected profit from a vacancy* is equal to the probability of filling the vacancy, and earning the difference between the present value of a job and the present value of the vacancy, minus the maintenance cost of the vacancy \( p_c \). It is defined by,

\[
q(\theta)(J - V) - p_c
\]  
(16.13)

Assuming a perfect capital market and an infinite time horizon, the present value of expected net profits from a vacancy \( V \) is defined by,

\[
V = \int_0^\infty e^{-rt} (q(\theta)(J - V) - p_c) dt = \frac{q(\theta)(J - V) - p_c}{r}
\]  
(16.14)

Thus, from (16.14), \( V \) must satisfy the Bellman equation,

\[
rV = -p_c + q(\theta)(J - V)
\]  
(16.15)

A vacancy is an asset for the firm. In a perfect capital market, the instantaneous expected net yield of this asset, \( q(J-V) - p_c \), will be equal to its expected opportunity cost, which is equal to \( rV \).

### 16.4.3 Free Entry and the Job Creation Condition

In equilibrium with free creation of vacancies (free entry), all profit opportunities by creating new vacancies will be exploited, and the expected profits from the creation of an additional vacancy will be equal to zero. So in equilibrium with free entry, \( V = 0 \). This, from (16.15), implies that,

\[
J = \frac{p_c}{q(\theta)}
\]  
(16.16)

This is an important prediction of this model. In equilibrium with free entry, the present value of a job will be equal to the expected cost of hiring an employee. This is equal to the instantaneous cost \( p_c \) of maintaining a vacancy, times the expected duration of the vacancy \( 1/q(\theta) \). Thus, competition for the creation of vacancies and free entry reduces the present value of profits from a job to the level of the expected cost of hiring a worker.

(16.16) and (16.12), imply that the marginal job must satisfy,

\[
w = p - \frac{(r + \lambda)p_c}{q(\theta)} = \left(1 - \frac{(r + \lambda)c}{q(\theta)}\right)p
\]  
(16.17)

(16.17) is the second key equation of this model, and can be called the *job creation condition*. The firm will only hire a worker and create a new job if the real wage is smaller than or equal to the productivity of the worker, minus the *marginal hiring cost*, which is defined as,

\[
\frac{(r + \lambda)p_c}{q(\theta)}
\]  
(16.18)
The marginal hiring cost is equal to the gross opportunity cost of maintaining a job. It is the opportunity cost of the expected net present value of profits from the job, which, because of free entry, is equal to the expected cost of hiring an employee. The higher is labor market tightness $\theta$, and the higher is $c$, the higher the marginal hiring cost, as the expected duration and the cost of a vacancy, is higher.

(16.17) corresponds, for this model, to the usual condition for hiring in competitive models without hiring costs, which is non other that the real wage must be equal to (marginal) productivity. One can confirm that, by setting the cost of maintaining a vacancy $c$ equal to zero. In this case, the real wage is equal to average and marginal productivity $p$. In the general case, where maintaining a vacancy is costly, the firm will only hire a new worker if the marginal cost of maintaining the vacancy is recouped.

The job creation condition implies a negative relation between labor market tightness and the real wage. The higher is labor market tightness, the higher the expected duration of a vacancy, and the higher the marginal cost of maintaining a vacancy. Thus, the real wage that the firm would be prepared to pay, relative to productivity, will be lower. The job creation condition is depicted diagrammatically in Figure 16.2. It is negatively sloped and convex to the origin, because of the properties of the matching function.

### 16.5 The Behavior of Unemployed Job Seekers

(16.8) and (16.17) do not suffice in order to determine the three endogenous variables $u$, $w$ and $\theta$. In order to determine the real wage, we must now examine the behavior of prospective workers, i.e the unemployed job seekers.

The typical worker earns a real wage $w$ when employed, and is looking for a job when unemployed. For the duration of search, she has an instantaneous real income $z$, which depends on unemployment benefits and any other income or benefit from the use of free time. For simplicity we shall call $z$ the unemployment benefit.

We define as $U$ and $W$ the corresponding present values of expected income for an unemployed and an employed worker.

An unemployed job seeker has an instantaneous real income equal to $z$ and an instantaneous probability of finding a job equal to $\theta q(\theta)$. As a result, the present value of her expected income is defined by,

$$U = \int_{t=0}^{\infty} e^{-\gamma t} (z + \theta q(\theta)(W - U)) dt = \frac{z + \theta q(\theta)(W - U)}{r}$$

The permanent income of an unemployed person is equal to $rU$.

An employed worker has an instantaneous real income $w$, the real wage, but at each instant also faces the risk of losing her job, with probability $\lambda$. Therefore, the present value of her expected income $W$ is equal to,
The permanent income of an employed worker is equal to \( rW \).

Equations (16.19) and (16.20) can be solved for the permanent income of the unemployed and the employed as,

\[
W = \int_{t=0}^{\infty} e^{-r t} (w + \lambda(U - W)) dt = \frac{w + \lambda(U - W)}{r} \tag{16.20}
\]

(16.20)

The permanent income of an employed worker is equal to \( rW \).

Equations (16.19) and (16.20) can be solved for the permanent income of the unemployed and the employed as,

\[
rU = \frac{(r + \lambda)z + \theta q(\theta)w}{r + \lambda + \theta q(\theta)} \tag{16.21}
\]

\[
rW = \frac{\lambda z + [r + \theta q(\theta)]w}{r + \lambda + \theta q(\theta)} \tag{16.22}
\]

It is easy to see from (16.21) and (16.22) that if \( w \geq z \), then \( W \geq U \), and the permanent income of an employed worker cannot be lower than the permanent income of an unemployed job seeker. In what follows, we shall assume that \( w \) exceeds \( z \), which indeed turns out to be the case in equilibrium.

If \( w \) exceeds \( z \), no employed worker has an incentive to leave their job voluntarily and become unemployed, while all the unemployed workers would accept any job offer at the prevailing real wage. If the condition \( w > z \) is satisfied, unemployment is involuntary. The present value of the expected income of an unemployed job seeker is lower that the present value of the expected income of an employed worker. This holds, despite the fact that even the workers who are employed may, with some probability, lose their jobs and become unemployed in the future. Thus, unlike the competitive real business cycle unemployment is involuntary in this model, and the unemployed are worse off than the employed.

**16.6 Wage Bargaining and the Wage Equation**

An unfilled vacancy implies a lower expected net present value of profits than a job, despite the fact that, with some probability, a job may disappear. Unemployment implies a lower permanent income for those who experience it, than for employed workers, despite the fact that even the workers who are employed may, with some probability, lose their jobs and become unemployed in the future.

A firm with a vacancy will hire an unemployed job seeker if the job creation condition is satisfied, i.e if the real wage is lower than or equal to productivity minus the marginal hiring cost. An unemployed job seeker will accept a job offer if the real wage is higher than the unemployment benefit.

The real wage for a particular job is determined by a negotiation between a prospective employer, a firm with a vacancy, and a prospective employee, an unemployed job seeker. Because all jobs are equally productive and all the unemployed receive the same unemployment benefit, the real wage that is determined by an individual negotiation will be the same as the real wage that prevails in the rest of the economy.

From the assumptions that we have made about productivity and aggregate disturbances, any one employer and any one worker, when they get together through the matching process, will certainly
agree to an employment contract and create a job. Otherwise they must continue searching, with additional costs for both sides.

An employment contract between an employer and an employee is defined by a real wage and the provision that employment will be terminated if there is a disturbance that makes the job untenable.

For a real wage \( w_i \), the expected return for a prospective employer and a prospective employee are given by,

\[
\begin{align*}
    rJ_i &= p - w_i - \lambda (J_i - V) \\
    rW_i &= w_i + \lambda (U - W_i)
\end{align*}
\]  

In (16.23) we have not made use of the assumption that competition has reduced the expected present value of a vacancy \( V \) to zero. We have assumed though, as it appropriate, that the expected present value of a vacancy, and of the income of the unemployed \( U \) depends on real wages in the rest of the economy, and is thus independent of \( i \).

The real wage is determined by a (generalized) Nash bargain, that maximizes the weighted product of the surplus of the prospective employer and the prospective employee from the agreement to create a job. An agreement means that the prospective employer gets a surplus \( J_i - V \) and the prospective employee a surplus \( W_i - U \). Thus, the real wage will satisfy,

\[
w_i = \arg \max (W_i - U)^\beta (J_i - V)^{1-\beta}, \quad 0 \leq \beta \leq 1
\]

\( \beta \) is a measure of the relative bargaining power of the prospective employee, over and above what results from the “threat” points \( U \) and \( V \).

The first order condition for the maximization of (16.25) implies that,

\[
W_i - U = \beta (J_i + W_i - V - U)
\]

Thus, in this model, \( \beta \) turns out to be the share of the prospective employee in the total surplus generated by the creation of a new job.

Using (16.23) and (16.24), and imposing the condition \( V = 0 \), we get,

\[
w_i = rU + \beta (p - rU)
\]

From (16.27), the employee gets a real wage which exceeds the permanent income of an unemployed worker by a multiple \( \beta \) of the difference of productivity from the permanent income of an unemployed worker.

From (16.27) it is obvious that real wages will be the same for all workers, as the right hand side does not depend on \( i \).

---

\[ ^8 \] In a symmetric bargain, as the one we analyze, a reasonable value of \( \beta \) would be \( 1/2 \) but it is not necessary to assume this particular value.
From (16.26) and (16.16), for \( V=0 \), we can drop \( i \), and get,

\[
W - U = \frac{\beta}{1-\beta} J = \frac{\beta pc}{(1-\beta)q(\theta)} \tag{16.28}
\]

From (16.19) and (16.28),

\[
rU = z + \theta q(\theta)(W - U) = z + \frac{\beta}{1-\beta} pc\theta \tag{16.29}
\]

Using (16.29) to substitute for \( rU \) in the wage function (16.27),

\[
w = (1-\beta)z + \beta p(1+\theta) = z + \beta(p-z) + \beta pc\theta \tag{16.30}
\]

(16.30) is the most convenient and easily interpretable form of the wage determination function in this model. \( p \) is labor productivity and \( pc\theta \) is the average recruitment cost per unemployed worker. The real wage exceeds the unemployment benefit \( z \). It exceeds it by a proportion \( \beta \) of the difference between productivity and the unemployment benefit, plus a proportion \( \beta \) of the average recruitment cost per unemployed worker.

Higher labor market tightness \( \theta \) results in a higher real wage, as this increases the average recruitment cost per unemployed person, thereby increasing the “threat” point of prospective employees versus prospective employers, and weakening the effective bargaining position of prospective employers.

It is worth noting that while the job creation condition (16.12) plays a role analogous to that of a labor demand function in competitive labor market models without hiring costs, the wage function (16.24) plays the role of a labor supply function. Both are needed in order to determine real wages and labor market tightness.

### 16.7 Wage Determination and Equilibrium Unemployment

In steady state this model determines the three endogenous variables, \((u, \theta, w)\) which simultaneously satisfy the Beveridge curve, i.e. the condition of equality of flows in and out of unemployment (16.8), the job creation condition (16.17) and the wage equation (16.30).

\[
u = \frac{\lambda}{\lambda + \theta q(\theta)} \tag{16.8}
\]

\[
w = \left(1 - \frac{(r + \lambda)c}{q(\theta)}\right)p = 0 \tag{16.17}
\]

\[
w = (1-\beta)z + \beta p(1+\theta) \tag{16.30}
\]
We can see from (16.17) and (16.30), that we can determine the level of real wages $w$ and labor market tightness $\theta$, independently of the unemployment rate $u$. The job creation condition and the wage equation suffice for the determination of real wages and labor market tightness. Once we determine labor market tightness $\theta$, we can substitute for it in the Beveridge curve (16.8), and determine equilibrium unemployment.

The determination of equilibrium is presented in Figure 16.3. The wage equation (16.30) implies a positive relation between the real wage and labor market tightness. The job creation condition (16.17) implies a negative relation between the real wage and labor market tightness. Equilibrium real wages and labor market tightness are determined at the intersection of these two curves.

The wage function has a positive slope because the higher is labor market tightness, the higher is the wage that prospective employees can negotiate with employers. The job creation condition has a negative slope because higher wages make creating vacancies less profitable, and thus reduce vacancies relative to unemployment. We also see that the determination of real wages and labor market tightness do not depend on the unemployment rate. This is because of the assumption of constant returns to scale in the matching function.

Once $\theta$ is determined from the wage equation and the job creation condition, we can substitute in the Beveridge curve (16.8) and determine for the unemployment rate. The solution can be shown diagrammatically using Figure 16.1.

In Figure 16.1, the positively sloped straight line has a slope of $\theta$, the ratio of vacancies to the unemployed. This is endogenously determined in equilibrium, as analyzed in Figure 16.1. The negatively sloped Beveridge curve is derived from equation (16.8), the equality of flows in and out of unemployment. Higher vacancies imply lower unemployment, because the probability of finding a job by an unemployed job seeker is higher. The curve is convex to the origin, because of the properties of the matching function. The equilibrium unemployment rate is determined at the intersection of the Beveridge curve with the straight line through the origin, the slope of which is determined by labor market tightness.

### 16.8 Implications of Shifts in the Parameters of the Model

We can now use the full model in order to examine how real wages, labor market tightness and equilibrium unemployment depend on the various exogenous parameters.

#### 16.8.1 Implications of an Increase in Labor Productivity

An increase in labor productivity is an increase in $p$. This will shift both the job creation condition to the right and the wage equation upwards as shown in Figure 16.4. Since $\beta < 1$, the shift in the job creation condition is greater, and in the new equilibrium $A$, both real wages and labor market tightness will rise. As a result, equilibrium unemployment will fall.

This effect of labor productivity occurs because of the assumption that the unemployment benefit $z$ is fixed and does not depend on labor productivity. In this case, changes in real wages do not fully reflect changes in labor productivity. Thus, when productivity increases, there are gains from the creation of new job vacancies, new vacancies are created and unemployment falls.
However, in many countries unemployment benefits $z$ are not fixed in real terms, and usually are related to the real wages of the employed. Thus, the assumption of an exogenously fixed unemployment benefit $z$ is not satisfactory.

Moreover, if the prediction that the growth of labor productivity reduces the unemployment rate was correct, then unemployment would be tending towards zero in the long run, because of the continuous increases in labor productivity resulting from economic growth. This prediction is not compatible with the existing empirical evidence, which which does not suggest any long run trends for unemployment rates.

Assuming that unemployment benefits $z$ are a percentage $\rho$ of real wages, where $0<\rho<1$, then in our model we would have that $z=\rho w$. $\rho$ is usually called the replacement rate. Labor legislation in most industrialized countries provides for an unemployment benefit which is a percentage of the wage received by the unemployed person in his last job, or a percentage of the minimum wage. Therefore, this assumption is more satisfactory and realistic than the assumption of an exogenously determined real unemployment benefit.

In this case, the wage function (16.30) is converted to,

$$w = \frac{\beta(1 + c\theta)}{1 - (1 - \beta)\rho} p$$  \hspace{1cm} (16.31)

With the assumption of an exogenous replacement rate, the wage equation (16.31) becomes proportional to labor productivity $p$. The factor of proportionality depends positively on the bargaining power of prospective employees $\beta$, labor market tightness $\theta$, the cost of maintaining a vacancy $c$, and the replacement rate of the unemployment benefit system $\rho$. An increase (or fall) of labor productivity, for given labor market tightness, results in an equiproportional shift in the wage equation.

From the job creation condition (16.12), a rise in productivity also results in an equiproportional shift in the real wage, for given labor market tightness.

This case is also presented in Figure 16.4. An increase in labor productivity moves the wage equation and the job creation condition by the same proportion, thus increasing real wages without affecting the degree of labor market tightness. Consequently, the unemployment rate would not be affected by an increase in labor productivity either.

Another way to see the independence of labor market tightness from labor productivity when the unemployment benefit is proportional to the real wage, is to equate the right hand sides of (16.12) and (16.25). This results in,

$$1 - \frac{(r + \lambda)c}{q(\theta)} = \frac{\beta(1 + c\theta)}{1 - (1 - \beta)\rho}$$ \hspace{1cm} (16.32)

(16.32) implies that in equilibrium, labor market tightness is independent of labor productivity. As a result, the unemployment rate will also be independent of labor productivity.
The independence of labor market tightness and therefore the equilibrium unemployment rate from average labor productivity is a desirable property, as unemployment rates do not display a long-term trend, as would be the case if they depended on increases in labor productivity which have an upward long term trend.

16.8.2 Implications of a Rise in Unemployment Benefits

One can also examine the implications of an increase in unemployment benefits $z$, or, in the case, $z=\rho w$, an increase in the replacement rate $\rho$. This causes an increase in real wages, reduces labor market tightness and results in higher equilibrium unemployment.

The relevant analysis is in Figure 16.5. An increase of $z$ (or $\rho$ in the case $z=\rho w$), shifts the wage equation upwards, as prospective workers demand higher wages given that the cost of unemployment is now lower. With higher wages firms create fewer jobs through the job creation condition. Consequently, the unemployment rate increases and the proportion of vacancies decreases. An increase of $\beta$, the relative bargaining power of job seekers, would have the same impact for similar reasons. Real wages rise and labor market tightness is reduced. Consequently, the unemployment rate rises and the vacancy rate decreases.

16.8.3 Implications of a Rise in the Real Interest Rate

In Figure 16.6 we analyze the impact of an exogenous increase in the real interest rate. An increase in the real interest rate shifts the job creation condition in Figure 16.5 to the left, as it increases the cost of maintaining vacancies. This results in a reduction of both real wages and labor market tightness. The reason is that with a higher real interest rate, expected future revenues from the creation of a new job have a lower present value, while the cost of creating the new job is paid up front. Therefore there is less of an incentive to create new jobs. The fall in labor market tightness naturally leads to an increase in the unemployment rate through the Beveridge curve.

16.8.4 Implications of an Increase in the Probability of Termination of a Job

We finally analyze the implications of an increase in $\lambda$, the exogenous probability of termination of a job. This case is analyzed in Figure 16.7.

An increase in $\lambda$ leads to a reduction in real wages and labor market tightness, because it reduces the expected revenue from the creation of a new job. Consequently, fewer job vacancies are created, labor market tightness is reduced and flows from unemployment to jobs decrease. This leads to an increase in the unemployment rate.

In this case, equilibrium unemployment also increases because flows into unemployment also rise. An increase in $\lambda$ shifts the Beveridge curve to the right, as flows from existing jobs to unemployment rise. Consequently, equilibrium unemployment increases both because of the increase of flows from employment to unemployment, resulting directly from the rise of $\lambda$, and because of the decrease of flows from unemployment to jobs, as a result of the fall in labor market tightness. The impact of an increase in $\lambda$ on the equilibrium vacancy rate is uncertain, as it depends on the exact values of the parameters of the model.
Thus, the theoretical analysis implies an increase in equilibrium unemployment following a rise in unemployment benefits (or the replacement ratio), real interest rates and the probability of termination of a job. The reason is that all these shocks reduce labor market tightness and the vacancy rate, reducing flows from unemployment to jobs. In addition, an increase in the probability of termination of a job also increases flows from jobs into unemployment, further increasing equilibrium unemployment.

16.9 Dynamic Adjustment to the Steady State

Our analysis so far was only concerned with steady state unemployment, and almost nothing was said about the dynamic adjustment of the labor market in the short run. The dynamic adjustment of this model is analyzed in Pissarides (1985, 2000).

Assuming that vacancies and real wages are non predetermined variables in the short run, since they depend on forward looking expectations of firms and job seekers, and that unemployment is a predetermined variable, governed by equation (16.6), one can show that there is a unique saddle path that leads unemployment to its steady state value.

16.9.1 The Dynamic Adjustment of Unemployment and Vacancies

From (16.6), the unemployment rate evolves according to,

\[ u(t) = \lambda - (\lambda + \theta_E q(\theta_E))u(t) \]  \hspace{1cm} (16.33)

where \( \theta_E \) is the steady state labor market tightness determined through the wage negotiations of firms with vacancies and unemployed job seekers analyzed in the previous sections. Since vacancies are a non predetermined variable, and \( \theta \) is determined independently of the unemployment rate, \( \theta \) and real wages \( w \) jump immediately to their steady state values.

From the solution of the differential equation (16.33), the short run evolution of unemployment is determined by,

\[ u(t) = u_E + (u_0 - u_E)e^{-(\lambda + \theta_E q(\theta_E))t} \]  \hspace{1cm} (16.34)

where \( u_0 \) is the unemployment rate at time \( 0 \), and \( u_E \) is the steady state unemployment rate, defined as,

\[ u_E = \frac{\lambda}{\lambda + \theta_E q(\theta_E)} \]  \hspace{1cm} (16.35)

From (16.34) the unemployment rate will converge to its steady state value from any initial rate, with a speed of adjustment equal to \( \lambda + \theta_E q(\theta_E) \). Thus, the model predicts a stable and unique adjustment path for unemployment, following any shock that changes the steady state (“natural”) unemployment rate.\(^9\)

\(^9\) Note that for the parameter values used in our reference simulation in the next section, the speed of adjustment is equal to 0.49, implying that it takes approximately 1 year to close half of the gap between current and equilibrium unemployment.
Since $\theta_E$ remains constant during the adjustment path, vacancies will be adjusting at the same rate as unemployment along the adjustment path, implying that,

$$v = \theta_E u$$  \hspace{1cm} (16.36)

The relevant phase diagram in vacancy-unemployment space is shown in Figure 16.8. We assume that the labor market is originally in steady state equilibrium $(v_0,u_0)$, and there is an unanticipated permanent shock that changes $\theta$ from $\theta_0$ to $\theta_E$. We assume that $\theta_E$ implies a higher steady state unemployment rate $u_E$, and a lower steady state vacancy rate $v_E$. How will the labor market adjust?

As can be seen from the phase diagram in Figure 16.8, vacancies, which are the non-predetermined variable, will jump from $v_0$ to $v_1$. There will be an immediate drop in vacancies to ensure that labor market tightness jumps to its new steady state value. Following that, both the unemployment rate and the vacancy rate will be gradually adjusting upwards towards their new steady state values, as described by the differential equations (16.33) and (16.36). It is because the unemployment rate is a predetermined variable, that the non predetermined vacancy rate initially overshoots its steady state drop.

Thus, this model explains not only the steady state unemployment rate, but also the gradual dynamic adjustment of the unemployment rate towards its steady state, following shocks to the determinants of equilibrium wages and equilibrium tightness in the labor market.

The model has been extended and applied and tested extensively. Merz (1995) and Andolfatto (1996) have introduced the model into a real business cycle framework, with intertemporal substitution of leisure and capital accumulation. Blanchard and Diamond (1989, 1990) and Shimer (2005), among others, have estimated and tested the model for US data, with generally mixed results.

16.9.2 Numerical Simulations of the Model

In order to get a quantitative sense of the extent to which shifts in the various parameters affect the equilibrium unemployment rate, real wages and labor market tightness, it is worth simulating the model numerically.

In this simulation we assume that the matching function is Cobb Douglas with constant returns to scale, and is described by,

$$mL = M(uL)^\mu (vL)^{1-\mu}$$  \hspace{1cm} (16.37)

where $M>0$ is the efficiency of the matching process and $0<\mu<1$ the elasticity of new jobs with respect to unemployment.

From (16.37), the probability of filling a vacancy is determined by,

$$q(\theta) = \frac{mL}{vL} = M \left( \frac{u}{v} \right)^\mu = M \theta^{-\mu}$$  \hspace{1cm} (16.38)
Substituting (16.38) in the Beveridge curve (16.8) and the job creation condition (18.12), the model takes the form,

\[
\begin{align*}
u &= \frac{\lambda}{\lambda + M\theta^{1-\mu}} \tag{16.8'} \\
w &= p - \frac{(r + \lambda)pc}{M\theta^{-\mu}} \tag{16.17'} \\
w &= (1 - \beta)z + \beta p(1 + c\theta) = (1 - \beta)pw + \beta p(1 + c\theta) \tag{16.30}
\end{align*}
\]

We assume the following initial parameter values: \(\lambda=2.5\%, M=1/2, \mu=1/2, p=1, c=1/2, r=3\%, \rho=1/2, \beta=1/2\).

Simulating the model for these parameter values, we get \(w=0.95, \theta=0.85, u=5.1\%, v=4.4\%\).

The real wage is 95% of productivity and the unemployment rate is relatively low at 5.1%.

If the probability of termination of a job \(\lambda\) were to double to 5%, then it follows that \(w=0.93, \theta=0.79, u=10.1\%, v=7.9\%\).

Relative to the original equilibrium, the real wage falls by 2.1%, to 93% of productivity, and the steady state unemployment rate almost doubles, to 10.1%.

The dynamic adjustment of the unemployment rate and the vacancy rate is depicted in Figure 16.9. The vacancy rate initially falls, and then starts rising as the unemployment and vacancy rates gradually rise towards their higher steady state values.

If the replacement ratio \(\rho\) were to rise from 0.5 to 0.7 (a 40% increase), then it follows that \(w=0.96, \theta=0.50, u=6.6\%, v=3.3\%\).

Relative to the original equilibrium, the real wage rises by roughly 1%, and the unemployment rate rises by 1.5 percentage points, to 6.6%. With a constant labor force, this is equivalent to a 29.4% rise in the number of the unemployed.

The dynamic adjustment of the unemployment rate and the vacancy rate is depicted in Figure 16.10. Note the overshooting of the fall of the vacancy rate, relative to its steady state fall, which conforms with the theoretical analysis in Figure 16.8. Because of the fall in equilibrium labor market tightness, the vacancy rate initially falls. As the unemployment rate gradually rises towards its higher new steady state value, the vacancy rate gradually rises as well. Thus, the short run reduction in the vacancy rate exceeds the steady state reduction.

Finally, if the real interest rate \(r\) were to double from the original 3% to 6%, it follows that \(w=0.925, \theta=0.78, u=5.4\%, v=4.2\%\).
Relative to the initial equilibrium, the real wage falls by about 2.5%, and the unemployment rate rises by 0.3 percentage points, to 5.4%. With a constant labor force, this is equivalent to a 5.9% rise in the number of the unemployed.

The dynamic adjustment of the unemployment rate and the vacancy rate is depicted in Figure 16.11. Note again the overshooting of the fall of the vacancy rate.

We see from these simulations, and for the specific values of the parameters, that shifts in the probability of terminating existing job, as well as shifts of unemployment benefits, have relatively large effects on the equilibrium unemployment rate and relatively small effects on real wages. On the other hand, changes in the real interest rate have a relatively modest effect on both real wages and the unemployment rate.

16.10 Conclusions

In this chapter we have examined the determination of equilibrium unemployment in a matching model on the labor market.

In this model, employers are investing in order to create job vacancies and the process of creating jobs involves matching of firms with vacancies with unemployed job seekers.

At each instant, there are two flows into and out of unemployment. Some workers lose their jobs and move from jobs into unemployment, and some of the unemployed find jobs, through the matching process, with firms with vacancies.

In the simpler versions of the model the probability of termination of a job is exogenous. This parameter describes the structural or cyclical shocks affecting the economy, and leading to the destruction of jobs.

The probability of filling a vacancy, as well as the probability of an unemployed job seeker to find a job, are endogenous variables in this model. They depend on the degree of labor market tightness, which is defined by the ratio of vacancies to the unemployed. The higher the tightness of the labor market, the greater the probability of an unemployed job seeker to find a job, and the lower the probability of a firm to fill a vacancy.

In the steady state, the flows to and from unemployment are equalized, and the equilibrium unemployment rate depends positively on the exogenous probability of termination of a job, and negatively on the endogenous probability of an unemployed job seeker to find a job. The equilibrium unemployment rate therefore depends negatively on labor market tightness, and is, of course, determined endogenously. The negative relationship between the equilibrium unemployment rate and the vacancy rate which is implied by this dependence is known as the Beveridge curve.

Firms and the unemployed make their decisions rationally, maximizing the expected present value of their profits and income.

Firms create new vacancies as long as the expected profits from the investment required to create a vacancy are positive. The condition for a vacancy to be filled, and for a new job to be created is that
the real wage should be equal to labor productivity minus the cost of creating and maintaining a vacancy. By filling a vacancy, a firm must in equilibrium cover both the wage costs and the costs of its investment in the creation of the vacancy.

The job creation condition implies a negative relationship between the wage that the firm is willing to pay and labor market tightness. The higher is labor market tightness, the lower is the probability of filling a vacancy and the greater the total cost of maintaining a vacancy, since vacancies remain unfilled for longer.

On the other hand, an unemployed job seeker will agree to get a job if the expected present value of income of an employed worker is greater than the expected present value of income of an unemployed job seeker. This condition is satisfied in this model, as long as the real wage is higher than unemployment benefits.

Real wages are determined in equilibrium by decentralized bargaining between firms that have vacancies and unemployed job seekers. The equilibrium real wage is the result of this negotiation, and depends positively on the relative bargaining power of the unemployed, the level of unemployment benefits, labor productivity, the cost of maintaining a vacancy and labor market tightness. The equilibrium real wage depends positively on labor market tightness, as this increases the average recruitment cost per unemployed person, thereby increasing the “threat” point of prospective employees versus prospective employers, and weakening the effective bargaining position of prospective employers.

The positive relationship between real wages and labor market tightness, resulting from the negotiation between firms with vacancies and the unemployed, and the negative relationship between real wages and labor market tightness implied by the job creation condition for firms, jointly determine the equilibrium real wage and equilibrium labor market tightness. For given equilibrium labor market tightness, the equilibrium unemployment rate is then determined through the Beveridge curve, which implies a negative relationship between the unemployment and vacancy rates.

In the equilibrium of this model, the unemployed are worse off than the employed. Consequently unemployment is an undesirable and involuntary condition, and not the result of choice by the unemployed, as in competitive models of the labor market without frictions.

If unemployment benefits are proportional to the real wage, labor productivity does not affect the equilibrium unemployment rate in this model.

However, the higher the percentage of real wages paid out as unemployment benefits, the higher the equilibrium real wage and the higher the equilibrium unemployment rate. The reason is that higher real wages reduce incentives for creating new jobs, thus reducing the number of vacancies, reducing labor market tightness and increasing unemployment.

Higher real interest rates also have a positive impact on unemployment in this model, because they increase the cost of maintaining a vacancy, resulting in the creation of fewer job vacancies, lower labor market tightness and higher unemployment.
A higher exogenous probability of termination of a job has a positive impact on unemployment for two reasons. First because it directly increases the flows from existing jobs to unemployment, and, second, because it, indirectly, reduces the flows from unemployment to jobs. The second effect takes place because the expected profit from the creation and filling of a vacancy falls, resulting in fewer vacancies and reduced flows from unemployment to jobs.

In this model, equilibrium unemployment depends both on cyclical and structural factors. Moreover, unlike the new classical model of real economic cycles, but like some versions of the “new Keynesian” model, unemployment is “involuntary” in the sense that the unemployed would always prefer to be in jobs at prevailing real wages.

The adjustment path predicted by the model is stable, as, following shocks to the determinants of equilibrium unemployment, the unemployment and vacancy rates follow a unique and stable adjustment path towards the new equilibrium.
Figure 16.1
Unemployment, Vacancies and the Beveridge Curve

![Diagram showing the Beveridge curve and labor market tightness. The curve intersects the vertical axis at $v_E$ and the horizontal axis at $u_E$. The point of intersection is labeled E. The diagram explains the relationship between employment and vacancies.]
Figure 16.2
The Job Creation Condition
Figure 16.3
The Determination of Real Wages and Labor Market Tightness

The diagram illustrates the determination of real wages and labor market tightness. It shows two curves: the Wage Equation and the Job Creation Condition. The point of intersection, E, represents the equilibrium real wage, $w_E$, and the labor market tightness, $\theta_E$. The Wage Equation is depicted as a downward-sloping curve, indicating that as the real wage increases, the quantity of labor supplied decreases. The Job Creation Condition curve shows the relationship between the real wage and the level of labor market tightness, with higher tightness requiring a lower real wage to encourage job creation.
Figure 16.4
Implications of an Increase in Labor Productivity
Figure 16.5
Implications of an Increase in Unemployment Benefits or the Replacement Rate
Figure 16.6
Implications of an Increase in the Real Interest Rate
Figure 16.7
Implications of an Increase in the Probability of Termination of a Job
Figure 16.8
Dynamic Adjustment of Unemployment and Vacancies
Figure 16.9
Dynamic Adjustment of Unemployment and Vacancies
Following an Increase in the Probability of Termination of a Job from 2.5% to 5%
Figure 16.10
Dynamic Adjustment of Unemployment and Vacancies
Following an Increase in the Replacement Rate from 50% to 70%
Figure 16.11
Dynamic Adjustment of Unemployment and Vacancies
Following an Increase in the Real Interest Rate from 3% to 6%
References


_American Economic Review_, 95, pp. 25-49.