Chapter 13
A Perfectly Competitive “New” Classical Model

In this chapter we introduce a perfectly competitive “new classical” model of aggregate fluctuations in which there is no capital accumulation. This is a dynamic stochastic general equilibrium models (DSGE), based on optimizing households and firms, flexible wages and prices, and fully competitive markets. Fluctuations are caused by real shocks to productivity, but we abstract from capital accumulation which propagates the dynamic effects of such shocks in the stochastic growth model of Chapter 11.

The reason for introducing this simplified model, is to have a “new classical” model which is comparable to the “new keynesian” models that we shall analyze in subsequent chapters, so that we can better understand the similarities and differences between the “new classical” and the “new keynesian” approaches to aggregate fluctuations.

The main impulses that generate aggregate fluctuations are real, i.e shocks to productivity. However, we also analyze the role of monetary shocks and monetary policy in this class of “new classical” models.

Monetary shocks have no real effects on output, employment and other real variables in this model, and only affect real money balances, and nominal variables such as the price level, inflation and nominal wages and interest rates. Thus, the role of monetary policy in this model is simply to stabilize inflation in the presence of monetary and real shocks.

13.1 A Perfectly Competitive “New Classical” Model without Capital

In what follows we shall focus on a perfectly competitive “new classical” model of aggregate fluctuations, in which the only variable factor of production is labor. We shall thus abstract from capital accumulation, which is an important propagation mechanism in the stochastic growth model that we analyzed in Chapter 11.

In this analytically simpler model we allow for a more general approach to the preferences of the representative household, and also distinguish between nominal and real variables, in order to consider the determination of the level of prices and wages, inflation and nominal interest rates, and the role of monetary factors in “new classical” models.

13.1.1 The Representative Household

The representative household is assumed to maximize,
where $C$ is consumption and $L$ is labor supply. We assume that,

$$u_C = \frac{\partial u}{\partial C_i} > 0, \quad u_{C,L} = \frac{\partial^2 u}{\partial C_i \partial L_i} \leq 0, \quad u_{L,L} = \frac{\partial u}{\partial L_i} \leq 0, \quad u_{L,L} = \frac{\partial^2 u}{\partial C_i^2} \leq 0.$$  (13.2)

The constraints under which the maximization takes place are given by,

$$P C_i + 1 \frac{1}{1+i} B_t \leq B_{t-1} + W_t L_t - T_t$$  (13.3)

$$\lim_{T \to \infty} E_T B_T \geq 0$$  (13.4)

where $P$ is the price level, $W$ the nominal wage, $i$ the nominal interest rate, $B$ a nominal one period bond, and $T$ an exogenous transfer of nominal income to the household (dividends, government transfers of taxes).

From the first order conditions it follows that,

$$-u_{L_t} = \frac{W_t}{P_t}$$  (13.5)

$$\frac{1}{1+i} = \frac{1}{1+\rho} E_t \left( \frac{u_{C_{t+1}} P_t}{u_{C_t} P_{t+1}} \right)$$  (13.6)

We assume that the per period utility function is given by,

$$U(C_t, L_t) = \frac{C_t^{1-\theta} L_t^{1+\lambda}}{1-\theta \frac{1}{1+\lambda}}, \quad \text{where } \theta > 0 \text{ and } \lambda > 0$$  (13.7)

The first order conditions for the problem of the representative household in this case take the form,

$$\frac{W_t}{P_t} = C_t^{\theta} L_t^\lambda$$  (13.8)

$$\frac{1}{1+i} = \frac{1}{1+\rho} E_t \left( \frac{C_{t+1}^{\theta}}{C_t} \frac{P_t}{P_{t+1}} \right)$$  (13.9)

(13.8) and (13.9) can be written in log-linear form, as,

$$w_t - p_t = \theta c_t + \lambda l_t$$  (13.10)
George Alogoskoufis, *Dynamic Macroeconomics*, 2016

\[ c_t = E_t(c_{t+1}) - \frac{1}{\theta}(i_t - E_t(\pi_{t+1}) - \rho) \]  

(13.11)

where \( w=\ln W, p=\ln P, c=\ln C, l=\ln L \) and \( \pi_t = p_t - p_{t-1} \) is the inflation rate.

### 13.1.2 The Representative Firm

Production of the representative firm is a positive function of employment, and is described by an aggregate production function of the form,

\[ Y_t = A_t L_t^{1-\alpha} \]  

(13.12)

where \( A>0 \) and \( 0<\alpha<1 \) are exogenous technological parameters. \( \alpha \) is a constant, while \( A \) follows an exogenous stochastic process.

The representative firm chooses employment in order to maximize profits, for given nominal wages and prices. Profits are determined by,

\[ P_t Y_t - W_t L_t \]  

(13.13)

Profit maximization implies that employment will be determined so as to equate the marginal product of labor to the real wage. Thus,

\[ \frac{W_t}{P_t} = (1-\alpha)A_t L_t^{1-\alpha} \]  

(13.14)

One can solve the marginal productivity condition for the price level. The interpretation is that the product price is equal to marginal cost.

\[ P_t = \frac{W_t}{(1-\alpha)A_t L_t^{1-\alpha}} \]  

(13.15)

Log-linearizing the first order condition (13.14) we get,

\[ w_t - p_t = a_t - \alpha l_t + \ln(1-\alpha) \]  

(13.16)

where \( a=\ln A \).

Log-linearizing the production function (13.12) we get,

\[ y_t = a_t + (1-\alpha)l_t \]  

(13.17)

Having determined the behavior of households and firms, we can now analyze the equilibrium in this model.

### 13.1.3 General Equilibrium
In the basic form of this model we shall assume that there is no investment or public consumption. Accordingly, in product market equilibrium, consumption will be equal to total output.

\[ y_t = c_t \]  \hspace{1cm} (13.18)

The equilibrium condition (13.18) will determine the real interest rate.

The condition for equilibrium in the labor market will require that labor demand, as implied by (13.16), should be equal to labor supply, as implied by (13.10). This will determine the real wage.

The model consists of equations (13.10), (13.11), (13.16) and (13.17) and the equilibrium condition (13.18), and determines employment, output, consumption, real wages and the real interest rate, as functions of the exogenous labor productivity process \( a \).

The real interest rate is defined by the Fisher equation, as,

\[ r_t = i_t - E_t(\pi_{t+1}) \]  \hspace{1cm} (13.19)

Solving the model for the five endogenous variables, we get,

\[ l_t = \eta_{LA} a_t + \bar{l} \]  \hspace{1cm} (13.20)

where, \( \eta_{LA} = \frac{1 - \theta}{\theta(1 - \alpha) + \alpha + \lambda} \) and, \( \bar{l} = \frac{\ln(1 - \alpha)}{\theta(1 - \alpha) + \alpha + \lambda} \).

\[ y_t = c_t = \eta_{YA} a_t + \bar{y} \]  \hspace{1cm} (13.21)

where, \( \eta_{YA} = 1 + (1 - \alpha)\eta_{LA} = \frac{1 + \lambda}{\theta(1 - \alpha) + \alpha + \lambda} \) and, \( \bar{y} = (1 - \alpha)\bar{l} \).

\[ w_t - p_t = \eta_{WA} a_t + \bar{\omega} \]  \hspace{1cm} (13.22)

where, \( \eta_{WA} = 1 - \alpha \eta_{LA} = \frac{\theta + \lambda}{\theta(1 - \alpha) + \alpha + \lambda} \) and, \( \bar{\omega} = (\theta(1 - \alpha) + \lambda)\bar{l} \).

\[ r_t = \rho + \theta \eta_{LA} E_t(\Delta a_{t+1}) \]  \hspace{1cm} (13.23)

(13.20), (13.21), (13.22) and (13.23), along with the product market equilibrium condition (13.18), determine the five endogenous variables, as a function of the exogenous labor productivity process \( a \).

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1 To quote from Fisher (1896), “When prices are rising or falling, money is depreciating or appreciating relative to commodities. Our theory would therefore require high or low interest according as prices are rising or falling, provided we assume that the rate of interest in the commodity standard should not vary.” (p. 58). The rate of interest in the commodity standard is the real interest rate, and rising or falling prices are expected inflation. The Fisher equation was further elaborated in Fisher (1930), where it was made even clearer that Fisher referred to expected inflation.
It worth noting that fluctuations in employment, output, consumption and real wages are a function only of fluctuations in labor productivity and fluctuations in the real interest rate depend on fluctuations in the expected rate of change of productivity.

Output, consumption and real wages are positive functions of productivity, while employment is a positive function of productivity only if $\theta<1$, i.e. if the intertemporal elasticity of substitution of consumption is greater than one. If $\theta>1$, employment is a negative function of productivity, while if $\theta=1$, employment is independent of productivity. This is because if $\theta<1$ the substitution effect dominates over the income effect, after a change in productivity and real wages, and employment rises. If $\theta>1$ the income effect dominates over the substitution effect, while in the case $\theta=1$ the two effects cancel each other out, and employment is not affected.

Only real factors, such as real productivity, affect fluctuations in real variables. As in the stochastic growth model, monetary factors such as money supply and nominal interest rates have no impact on the evolution of real variables.

13.2 Monetary Factors in the “New Classical” Model

In order to examine the impact of monetary factors in the “new” classical model, we shall assume the existence of a money demand function by households and firms, which, in logarithms, takes the form,

$$ m_t - p_t = y_t - \eta i_t \quad (13.24) $$

where $\eta$ is the semi-elasticity of money demand with respect to the nominal interest rate.

From the definition of the real interest rate through the Fisher equation (13.19), the nominal interest rate is equal to,

$$ i_t = r_t + E_t(\pi_{t+1}) \quad (13.25) $$

where the real interest rate $r$ is determined by (13.23) and is independent of monetary factors.

We will show that, like in the case of the models analyzed in Chapter 10, when the central bank follows a rule for the money supply, then the model determines the price level and the level of inflation and nominal interest rates. If the central bank pegs the nominal interest rate, the price level and the level of the nominal money supply cannot be determined, unless the nominal interest rate reacts sufficiently strongly to changes in the price level.

13.2.1 An Exogenous Path for the Money Supply

If the central bank determines an exogenous path for the money supply, then, from (13.24) and (13.25) it follows that,

$$ p_t = \frac{\eta}{1+\eta} E_t(p_{t+1}) + \frac{1}{1+\eta} m_t - \frac{1}{1+\eta} (y_t - \eta i_t) \quad (13.26) $$

Under the assumption that $\eta>0$, the solution of (13.26) implies that,
\[ p_t = \frac{1}{1 + \eta} \sum_{j=0}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^j E_t \left( m_{t+j} - y_{t+j} + \eta r_{t+j} \right) \]  \hspace{1cm} (13.27)

From (13.27), the price level and inflation are determined as functions of the exogenous path of the money supply, and the paths of real output and the real interest rate, which, as we have seen, are independent of monetary factors in the “new” classical model. The nominal interest rate is determined endogenously from (13.27) and (13.25).

### 13.2.2 An Exogenous Path for the Nominal Interest Rate

If we assume that the central bank follows an exogenous path for the nominal interest rate, then, from the Fisher equation (13.25), it follows that,

\[ E_t (\pi_{t+1}) = i_t - r_t \] \hspace{1cm} (13.28)

(13.28) does not determine inflation, but expected inflation, given the exogenous path of nominal interest rates. (13.28) is consistent with any price level path that satisfies,

\[ p_{t+1} = p_t + i_t - r_t + \xi_{t+1} \] \hspace{1cm} (13.29)

where \( \xi \) is any shock that satisfies \( E_t (\xi_{t+1}) = 0 \).

(13.29) suggests that there are multiple equilibria for the price level and inflation, depending on \( \xi \). This price level indeterminacy when the central bank follows an exogenous path for the nominal interest rate is also transferred to the money supply, through the money demand function (13.24). Consequently neither the money supply nor the price level can be determined uniquely when the central bank follows an exogenous path for the nominal interest rate.\(^2\)

However, not all interest rate rules result in price level indeterminacy. As suggested more than a century ago by Wicksell (1898), and we demonstrated in Chapter 10, if the central bank conditions its nominal interest rate on the price level, or inflation, then price level and inflation indeterminacy do not necessarily follow.

### 13.2.3 An Inflation Based Nominal Interest Rate Rule

Central banks predominantly use the nominal interest rate as their preferred monetary instrument. However, they follow policies according to which the path of nominal interest rates is not exogenous, but depends on past, current and expected future economic developments, mainly

\(^2\) The classic analysis of the appropriate choice of monetary instruments in a simple stochastic macro model was Poole (1970). As we argued in Chapter 10, Sargent and Wallace (1975) demonstrated that under rational expectations, a non-contingent interest rate target leads to price level indeterminacy and instability in such a model. However, it is now accepted that this problem does not arise in the case of contingent interest rate rules that make the nominal interest rate depend on the price level (McCallum 1981), or a sufficiently sensitive positive function of inflation. See Woodford (2003) Ch.1 for the relevant arguments. In addition, central banks, have been consistently using interest rates and not monetary aggregates as their main monetary policy instrument, especially in the last thirty years. As noted by Bernanke (2006), “In practice, the difficulty has been that, deregulation, financial innovation, and other factors have led to recurrent instability in the relationships between various monetary aggregates and other nominal variables.”.
inflation. For example, if inflation rises, central banks usually raise nominal interest rates in order to reduce it, and vice versa. This was after all the essence of the Wicksell rule. Let us therefore assume the following rule for determining nominal interest rates,

\[ i_t = \rho + \phi \pi_t \]  

(13.30)

where \( \phi > 0 \) is the reaction of the central bank nominal interest rate to inflation.

From (13.25) and (13.30) we therefore have that,

\[ \pi_t = \frac{1}{\phi} E_t(\pi_{t+1}) + \frac{1}{\phi}(r_t - \rho) \]  

(13.31)

where the real interest \( r \) depends only on real factors, as is the implication of “new” classical models.\(^4\)

Solving (13.31) under rational expectations,

\[ \pi_t = \sum_{s=0}^{\infty} \left( \frac{1}{\phi} \right)^s E_t(r_{t+s} - \rho), \text{ if } \phi > 1 \]  

(13.32)

\[ \pi_{t+1} = \phi \pi_t - (r_t - \rho) + \xi_{t+1}, \text{ if } \phi \leq 1 \]  

(13.33)

Thus, if the reaction of the central bank nominal interest rates to inflation is sufficiently pronounced (\( \phi > 1 \)), there is no indeterminacy problem for inflation. The fundamental solution is given by (13.33). If the reaction of the nominal interest rates to inflation is not sufficiently pronounced (\( \phi \leq 1 \)), then the problem of inflation indeterminacy and the possibility of “price bubbles” remains.

In any case, as we have already shown, in the “new” classical model of aggregate fluctuations only real factors affect fluctuations in real variables. Monetary factors and monetary policy only affect real money balances and nominal variables such as the price level and inflation, nominal interest rates and the nominal money stock.

### 13.2.4 Non-Neutrality of Money in “New Classical” Models

The short run neutrality of money implied by “new classical” models was initially troublesome for some of their proponents, as these models were not compatible with the evidence suggesting the existence of a positive short run relation between inflation and employment, i.e an expectations augmented Phillips curve.

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\(^3\) Wicksell (1898) was probably the earliest advocate of a such a stabilizing interest rate rule. We have shown how Wicksell’s rule results in price level determinacy in the models of Chapter 10. Here, we use a version of Wicksell’s rule for inflation rather than the price level.

\(^4\) See for example, equation (13.57). We shall further assume that the process determining the real interest rate is stationary.
Lucas (1972), responded by developing a “new classical” model which was consistent with a positive short run relation between inflation and employment. This model, which was subsequently implemented empirically by Lucas (1973) and Sargent (1973, 1976), was based on the assumption that firms did not have full information about the price level at the time they made their production decisions, and they attributed part of any change in the price level to a change in the relative price of their product. Thus, when inflation was unexpectedly high, all producers thought the relative price of their product had gone up, and thus increased production and employment. The opposite happened when inflation was unexpectedly low.²

However, this “new classical” explanation of the short run relation between inflation and output and employment was still not compatible with involuntary unemployment, and could only account for temporary deviations of output and employment from their “natural levels”, due to intertemporal substitution in labor supply and unanticipated inflation.

### 13.3 Conclusions

“New” classical models of aggregate fluctuations imply that aggregate fluctuations are caused by real factors. This is why such models are often called *real business cycle* models.

New classical models are *dynamic stochastic general equilibrium* models (DSGE) based on optimizing behavior by both households and firms, flexible prices, and fully competitive markets. Households maximize their intertemporal utility, firms maximize the present value of their profits, and markets function efficiently.

If the competitive general equilibrium models of this kind could explain all the features of aggregate fluctuations, then there would be no need for models that stress distortions in product and labor markets, and other market inefficiencies. However, “new” classical models have a number of weaknesses as models of aggregate fluctuations.

First, these models cannot account for the real effects of nominal and monetary shocks. For example, it is widely accepted that the Great Depression of the 1930s was caused by monetary and not real shocks. Similar views are prevalent regarding the recession of 2008-09, which was one of the deepest post World War II recessions. The sticky information assumption of Lucas (1972, 1973) can be used for accounting for the real effects of nominal shocks, but the effects of nominal shocks in such a model would be short lived and not persistent.

Second, even though “new” classical models can account for employment fluctuations, they only do so on the basis of intertemporal substitution in labor supply. This explanation, however, is not sufficient to explain the existence and the persistence of unemployment and the widely held view that unemployment is an involuntary condition for those who experience it, and not the result of a voluntary rational choice.

² A log linear version of the Lucas (1972) model was analyzed by Barro (1976), and was extended to incorporate the labor market and intertemporal substitution in labor supply by Alogoskoufis (1983). In the extended model, as workers could not observe the price level immediately, they systematically attributed part of unexpected inflation to relative price changes, and a temporary increase in their real wage. Thus, they increase labor supply in response to the increased labor demand of firms, and employment and output rise in response to unanticipated inflation. However, there is no unemployment in this model, and fluctuations in employment are based on intertemporal substitution in labor supply.
For these reasons, and despite the fact that “new” classical models are theoretically consistent, many economists consider them as an unsatisfactory explanation of aggregate fluctuations. The alternative class of models are “new keynesian” models, which assume that nominal wages and/or prices cannot adjust immediately in order to equilibrate labor and product markets. Thus, following nominal shocks, quantities have to adjust too, resulting in fluctuations in real variables, deviations of output and other real variables from their steady state values and involuntary unemployment. It is to such models that we now turn.
References


