Chapter 4
Models of Overlapping Generations

The representative household model is based on the assumption that all households are identical. An alternative class of models allows for households to differ. As young households are being born and old households die, there is a succession of overlapping generations. At any given time, households represent different generations. The savings behavior across generations generally differs, as households belonging to different generations may have differences in accumulated wealth and/or different time horizons. Thus, such models do not necessarily imply the homogeneity or economic efficiency that characterizes the basic representative household model.

The standard model in this category is the model of overlapping generations of Diamond (1965). This model is analyzed in discrete time, i.e., we assume that time is divided in discrete time periods rather than being a continuous variable. In each time period two types of households coexist. The young, who are in the first period of their lives, and the old, who are in the second and last period of their lives. Young households supply labor but own no capital. Thus, they only earn labor income. Old households are not working, and consume the income from the capital they accumulated during their first period of life, as well as their capital stock itself, since they live in the last period of life. In the following period, the old households have passed away, young households have become old, and a new generation of young households has entered the economy.

The production technology and the structure of markets is similar to that assumed in the Solow and Ramsey models. There are many competitive firms, the technology of production is described by a neoclassical production function, capital and labor markets are competitive, and capital and labor are paid their marginal product.

While the Diamond model is analyzed in discrete time, a more recent category of overlapping generations models is usually analyzed in continuous time (see Blanchard (1985) and Weil (1989)). In this more recent class of models, new households are being born continuously. All households, irrespective of their time of birth, have an infinite time horizon. In the original version of these models by Blanchard, at every instant there is also a constant probability of death, which is independent of the age of households. In the model of Weil, the probability of death is zero.

What emerges is that the differences of these overlapping generations models from a representative household model do no depend so much on the assumption of a positive probability of death, as in the Diamond and Blanchard models, but on the assumption that different generations have been

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1 The model of two overlapping generations of Diamond (1965) is based on a model of three overlapping generations, presented and analyzed by Samuelson (1958), to explain the demand for money as a store of wealth. For this reason it is often referred to as the Samuelson-Diamond model. We will analyze the Samuelson model in Chapter 10, where we analyze general equilibrium models with money.
born at different times in the past and thus hold different amounts of accumulated assets, which affect their savings behavior differently.

Savings behavior in overlapping generations models is not characterized by social efficiency, as in the representative household model. Moreover, in economies without a representative household, comparing utility across households is largely arbitrary. Furthermore, in the Diamond model, it is theoretically possible that the competitive equilibrium is not even Pareto efficient, as the possibility of dynamic inefficiency cannot be ruled out a priori. In any case, in models of overlapping generations, policy interventions that can improve social efficiency can be justified, since the competitive equilibrium is not necessarily optimal.

4.1 The Diamond Model

In the Diamond (1965) model, each household lives for only two periods. Thus, in each period, two overlapping generations of households coexist. The young, who are in their first period of life, and the old, who are in the second period of life. The young supply labor and earn labor income, while the old do not participate in the labor market, and live off their savings.2

4.1.1 Definitions

The definitions of the variables are analogous to the definitions we have used so far. As with the Solow and Ramsey models, we shall focus on the following set of endogenous variables:

\[ Y \] Aggregate Output (or \( y \), Output per efficiency unit of labor)
\[ K \] Aggregate (physical) Capital Stock (or \( k \), Capital per efficiency unit of labor)
\[ C \] Aggregate Consumption (or \( c \), Consumption per efficiency unit of labor)
\[ r \] Real interest rate
\[ wh \] Real wage per worker (or \( w \), Real Wage per efficiency unit of labor)

The exogenous variables and exogenous parameters in the model are defined as follows:

\[ t \] time (a discrete exogenous variable)
\[ L \] Population of the “young” generation (an exogenous variable that depends on time)
\[ h \] “efficiency” of labor (an exogenous variable that depends on time)
\[ n \] rate of growth of population (exogenous parameter)
\[ g \] rate of technical progress (exogenous parameter)
\[ \delta \] rate of depreciation of capital (exogenous parameter)
\[ \rho \] pure rate of time preference of households (exogenous parameter)

4.1.2 The Production Function

The technology of production is described by a neoclassical production function, analogous to the one we assumed in the Solow and Ramsey models. The production function has all the properties of the neoclassical production function assumed in the Solow model. The marginal product of all

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2 The time period in this model is equal to one half of the life expectancy of an adult, i.e about 30 years.
inputs is positive but decreasing, there are constant returns to scale and the Inada conditions are satisfied (see Chap. 1).

\[ Y_t = F(K_t, h_tL_t) \]  

Because of the assumption of constant returns to scale, the production function can be rewritten as,

\[ y_t = f(k_t) \]  

where,

\[ y = Y/hL \]  
\[ k = K/hL \]  
\[ f(k) = F(k, 1) \]

4.1.3 The Inter-temporal Utility Function of Households

In period \( t \), households are being born, and population grows at a rate \( n \). Thus,

\[ L_t = (1+n)L_{t-1} \]

The “young”, i.e those in the first period of life, work, while the “old”, those in the second period of life, do not work. Thus, each household supplies a unit of labor when “young” and allocates labor income between consumption of the first period and savings. In the second period of life, “old” households consume their savings, plus interest on their savings.

The utility function of a household born in period \( t \) is defined by,

\[ U_t = \frac{C_{1t}^{1-\theta}}{1-\theta} + \frac{1}{1+\rho} \frac{C_{2t+1}^{1-\theta}}{1-\theta}, \quad \theta > 0, \rho > -1 \]  

where,

\[ C_{1t} \]  
\[ C_{2t+1} \]  

(4.3) is the constant relative risk aversion or constant elasticity of substitution utility function, known to us from Chapter 2. \( \theta \) is the coefficient of relative risk aversion, and \( 1/\theta \) is the inter-temporal elasticity of substitution of consumption.

4.1.4 Markets and the Behavior of Households

Production takes place by many competitive firms, each with a production function such as (4.1). Capital and labor markets are competitive. Capital and labor are paid their marginal products, and, because of constant returns to scale, firms make zero profits. The real interest rate and the real wage are determined by,
where $f'(k)$ is the marginal product of capital. We have assumed for simplicity that the depreciation rate $\delta$ is equal to zero.

Each young household allocates its labor income $w t$ between consumption and savings. As a result, the capital stock of period $t+1$ is equal to the savings of period $t$.

$$K_{t+1} = L_t (w t h_t - C_{1t})$$

(4.5)

This capital stock is combined in production with the labor supplied by the generation born in period $t+1$, and the process continuous from period to period.

In the second period of life, the household born in period $t$, consumes all its interest income, plus the savings accumulated in the first period of life. Thus, consumption in the second period of life, of a household born in period $t$ is given by,

$$C_{2t+1} = (1 + r_{t+1})(w t h_t - C_{1t})$$

(4.6)

(4.6) implies that, for the household born in period $t$, the present value of consumption in periods $t$ and $t+1$, is equal to its labor income in period $t$.

$$C_{1t} + \frac{1}{1 + r_{t+1}} C_{2t+1} = w t h_t$$

(4.7)

Maximizing (4.3) under the budget constraint (4.7), and re-arranging the first order conditions, we get the following expressions for the ratio of consumption in the two periods,

$$\frac{C_{2t+1}}{C_{1t}} = \left( \frac{1 + r_{t+1}}{1 + \rho} \right)^{1/\theta}$$

(4.8)

(4.8) is analogous to the Euler equation for consumption in the Ramsey model. Consumption in the second period of life is higher or lower than consumption in the first period of life, depending on whether the real interest rate exceeds or falls short of the pure rate of time preference. $1/\theta$, the elasticity of inter-temporal substitution of consumption, determines the elasticity of the effect.

Using (4.7) and (4.8) to solve for $C_{1t}$ we get,

$$C_{1t} = \frac{(1 + \rho)^{1/\theta}}{(1 + \rho)^{1/\theta} + (1 + r_{t+1})^{(1-\theta)/\theta}} w t h_t$$

(4.9)

Young households consume a share of their labor income that depends on their preferences and the real interest rate. We can re-write (4.9) as,
$$C_t = (1 - s(r_{t+1}))w_t h_t$$

(4.9')

$s$ is the savings rate of young households, defined as,

$$s(r) = \frac{(1 + r)^{(1-\theta)/\theta}}{(1 + \rho)^{1/\theta} + (1 + r)^{(1-\theta)/\theta}}$$

(4.10)

(4.10) suggests that the savings rate is a positive function of the real interest rate $r$ only if $\theta$ is lower than unity, or if the elasticity of inter-temporal substitution of consumption $1/\theta$ is higher than unity. Only in this case the substitution effects dominates the negative income effect. If $\theta$ is greater than unity, the savings rate is a negative function of $r$; as the income effect dominates. In the special case that $\theta$ is equal to one (logarithmic preferences) the savings rate is independent of the real interest rate, and is equal to $1/(2+\rho)$.

We can now aggregate the savings behavior of the two overlapping generations and determine capital accumulation and the dynamic adjustment of this economy.

4.1.5 Capital Accumulation and the Dynamic Adjustment of the Economy

The aggregate capital stock in each period is equal to the savings of young households of the previous period, as the old households of the previous period consumed all their capital, and the young households of the current period own no capital. Thus,

$$K_{t+1} = s(r_{t+1})L_t w_t h_t$$

(4.11)

Expressing (4.11) in efficiency units of labor, dividing by $h_{t+1}L_{t+1}$, we get,

$$k_{t+1} = \frac{1}{(1 + n)(1 + g)} s(r_{t+1})w_t$$

(4.11')

Substituting for $r$ and $w$ from (4.4a) και (4.4b),

$$k_{t+1} = \frac{1}{(1 + n)(1 + g)} s(f'(k_{t+1}))(f(k_t) - k_t f'(k_t))$$

(4.12)

(4.12) determines the dynamic behavior of capital per efficiency unit of labor. Since output, the real interest rate, real wages and savings only depend on capital per efficiency unit of labour, (4.12) determines the dynamic evolution of all the other endogenous variables.

As with the previous models, the question that arises is whether a steady state equilibrium point $k^*$ exists, and if yes, whether the economy converges towards this equilibrium. (4.12) is a nonlinear first-order difference equation in the capital stock per efficiency unit of labor. Because of non-linearity, there is the possibility of multiple equilibria. Figure 4.1 highlights this possibility for a general form of the production function.

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3 These conditions are the same as the conditions discussed in the Ramsey model in Chapter 3.
The 45° line is the locus of equilibrium points, for which,

\[ k_{t+1} = k_t \]

The non-linear curve is the geometric version of a possible general form of (4.12). As shown in Figure 4.1, there are at least three possible equilibria for this general form of the curve, A, B and C. From these A and C are locally stable and B is unstable. Which equilibrium is reached by the economy depends on initial conditions.

If the initial capital per efficiency unit of labor is equal to \( k_0 \), then the economy will converge to equilibrium A. If the initial capital per efficiency unit of labor is higher than B, then the economy will converge to C. Thus, there are at least two potential stable equilibria, depending on initial conditions. There is no guarantee that the economy will converge to a unique equilibrium as in the Solow and Ramsey models.

Consequently, the Diamond model differs from the models examined so far, in that in its general form it can result in multiple equilibria. There is no unique equilibrium towards which an economy will converge regardless of initial conditions. Which equilibrium will prevail in the Diamond economy may thus depend on initial conditions. As illustrated in Figure 4.1, an economy with a low initial capital stock will converge to a balanced growth path like A, which is characterized by low capital per efficiency unit of labor. Per capita income in the balanced growth path will therefore be lower than in an economy which has a high initial capital stock (greater than that corresponding to point B), and which converges to a balanced growth path like C, which is characterized by a high capital stock per efficiency unit of labor, and a high per capita income on the balanced growth path.

Moreover, the equilibrium which eventually prevails may, under some conditions, depend on self-fulfilling expectations, or sunspots or the economy may be characterized by endogenous cyclical fluctuations, despite the fact that there are no exogenous random shocks. Balanced growth is therefore not guaranteed and the balanced growth path, if it exists, may depend on initial conditions.

### 4.1.6 The Diamond Model with Logarithmic Preferences and a Cobb Douglas Production Function

The Diamond model can be simplified significantly if one assumes logarithmic preferences (\( \theta = 1 \)) and a Cobb Douglas production function (\( f(k) = Ak^\alpha \)). Then, (4.10) and (4.12) are simplified to,

\[
s(r) = \frac{1}{2 + \rho} \quad \text{(4.10')}
\]

\[
k_{t+1} = \frac{1}{(1+n)(1+g)} \frac{1}{2 + \rho} (1 - \alpha)Ak^\alpha \quad \text{(4.12')}
\]

In this case, the savings rate is independent of the real interest rate. One can show, as in Figure 4.2, that in this case there is a unique balanced growth path, because of the properties of the Cobb Douglas production function. The Diamond economy converges to this unique balanced growth path independently of the initial conditions, as long as the initial capital stock is positive.
The properties of this simplified Diamond model, are analogous to the properties of the Solow and Ramsey models. On the unique balanced growth path the savings rate is constant. Output, consumption and the real wage per worker grow at a rate $g$, while the capital output ratio and the real interest rate is constant.

In Figure 4.3 we examine the impact of an increase in the pure rate of time preference in the simplified Diamond model. This has the effect of reducing the savings of the young generation, which leads to a lower level of capital in the following period compared with the initial equilibrium level $k^*$. This decline sets in motion a process of capital de-cumulation, until the economy converges to the new equilibrium B, in which the capital stock per efficiency unit of labor is lower. Consequently, as in the Ramsey model, an increase in the pure rate of time preference in the simplified Diamond model eventually leads to an equilibrium with a lower capital stock and real output and income per efficiency unit of labor.

An increase in the population growth rate $n$, or an increase in the rate of exogenous technical progress $g$ have similar effects to an increase in the pure rate of time preference, leading to a lower capital stock and real output and income per efficiency unit of labor.

A decrease in the pure rate of time preference has the opposite effects. In the new balanced growth path both consumption and capital per efficiency unit of labor will be higher. Consumption will initially fall, the economy will begin accumulating additional capital, and on the new balanced growth path the economy will end up with a higher capital stock and real output and income per efficiency unit of labor.

As in the representative household model, the pure rate of time preference is a key determinant of the savings rate. The higher it is, the smaller the savings of the young as well as the overall savings rate. Recall that in this model it is only the young that save, as the old have negative savings, since they consume both their current income and their capital.

In Figure 4.4 we analyze the impact of a previously unexpected permanent rise in total factor productivity $A$. This causes an immediate increase in income and savings for a given capital stock per efficiency unit of labor. The rise in aggregate savings leads to capital accumulation, until the capital stock approaches its new higher steady state value at $k^{**}$. Consequently, after a permanent increase in total factor productivity, the economy converges to a new balanced growth path, with higher steady state capital, output and income per efficiency unit of labor.

4.1.7 The Speed of Adjustment in the Simplified Diamond Model

As in the Solow and Ramsey models it is worth exploring the quantitative as well as the qualitative properties of the simplified Diamond model.

From (4.12'), the economy is on the balanced growth path $k^*$, when,

$$k^* = \frac{1}{(1+n)(1+g)} \frac{1}{2 + \rho} (1 - \alpha)A(k^*)^\alpha$$  \hspace{1cm} (4.13)

Solving for the steady state capital per efficiency unit of labor $k^*$ we get,
From the production function, steady state output per efficiency unit of labor \( y^* \) is then equal to,

\[
y^* = A(k^*)^\alpha = A^{\frac{1 - \alpha}{1(1 - \alpha)}} \left( \frac{(1 - \alpha)}{(1 + n)(1 + g)(2 + \rho)} \right)^{\alpha/(1 - \alpha)}
\]  

(4.15)

(4.15) determines output per efficiency unit of labor on the balanced growth path and its dependence on technological and population growth parameters, such as \( A, \alpha, g \) and \( n \), as well as parameters of the preferences of households, such as \( \rho \). From estimates of the values of the various parameters, we can get quantitative estimates for the long-term effects of various changes in the parameters on steady state output. It is worth noting that because in this model the length of the time period equals half the life expectancy of an adult, the choice of parameters should take account of this fact. For example, \( n \) must be calculated not as the annual rate of population increase, but as the population growth rate over 30 years. Similar choices have to be made for other parameters such as \( g \) and \( \rho \).

In order to find the speed of adjustment towards the balanced growth path, we get a linear approximation of \((4.12')\) around the steady state capital stock, \( k=k^* \). This is given by,

\[
k_{t+1} = k^* + \left( \frac{dk_{t+1}}{dk_t} \right)_{k=k^*} (k_t - k^*)
\]  

(4.16)

From \((4.12')\) we get that,

\[
\left( \frac{dk_{t+1}}{dk_t} \right)_{k=k^*} = \alpha \frac{(1 - \alpha)A}{(1 + n)(1 + g)(2 + \rho)} (k^*)^{\alpha - 1} = \alpha
\]

(4.17)

The speed of adjustment therefore depends on only one parameter, the exponent (share) of capital in the Cobb Douglas production function. If \( \alpha \) equals 1/3, then in each period the capital stock per efficiency unit of labor moves 2/3 of the distance that separates it from the value corresponding to the balanced growth path. If the period is 30 years, this amounts to about 2% a year, about as much as in the Solow model with a depreciation rate equal to zero.

Recall that in this model, under the assumption of logarithmic preferences, the savings rate is constant only for the young generation. The negative savings of the old generation are not a constant percentage of their income, but increase in absolute terms as the capital stock per efficiency unit of labor increases. This is because of diminishing returns to capital accumulation that reduce their interest income. As a result, in the simplified Diamond model, total savings as a percentage of total income (the aggregate savings rate) are a negative function of capital per efficiency unit of labor \( k \). The same applies to the representative household model with a Cobb Douglas production function and logarithmic preferences.
4.1.8 The Possibility of Dynamic Inefficiency

The greatest difference between the representative household model and the overlapping generations model is perhaps related to the efficiency of the equilibrium. We saw that in the Ramsey model competitive equilibrium leads to the maximization of welfare of the representative household. In overlapping generations models a representative household does not exist, and therefore it is not clear how we would evaluate the efficiency of the competitive equilibrium. If we define social welfare as a weighted average of the welfare of the different generations, there is no reason to expect that the competitive equilibrium would lead to the maximization of that weighted average.

The minimum efficiency criterion we have in economics is Pareto efficiency. Namely that it is impossible to make someone better off, without making somebody else worse off. But even the simplified Diamond model does not necessarily satisfy this criterion. Theoretically, the capital stock on the balanced growth path can exceed that which satisfy Pareto efficiency, and which corresponds to the golden rule. Therefore, a social planner could, if the economy ends up with more capital than the golden rule, theoretically increase the level of consumption and therefore the level of welfare of all generations, by reducing savings and the aggregate capital stock.

To see how this possibility can arise, we focus on the simplified Diamond model with logarithmic preferences and a Cobb Douglas production function.

The steady state capital stock is given by (4.14). Its marginal product is given by,

\[ f'(k^*) = \alpha A(k^*)^{\alpha - 1} = \frac{\alpha}{1-\alpha}(1+n)(1+g)(2+\rho) \]  \hspace{1cm} (4.18)

On the balanced growth path, the capital stock that satisfies the golden rule has a marginal product equal to the growth rate. It must satisfy,

\[ f'(k_{GR}) = aA(k_{GR})^{\alpha - 1} = (1+n)(1+g) - 1 \]  \hspace{1cm} (4.19)

For a sufficiently small \( \alpha \), the marginal product of the steady state capital stock, as given by (4.18), could be smaller than the marginal product corresponding to the golden rule, which is given by (4.19). That would mean that the steady state capital stock is higher than the golden rule capital stock, and a social planner could increase the current consumption and welfare of all generations, reducing savings and the steady state capital stock to the level corresponding to the golden rule. As we have already discussed in the case of the Solow model, a steady state in which the marginal product of capital is lower than the long-term growth rate of the economy (which defines the golden rule), is called dynamic inefficiency.

How likely is dynamic inefficiency in the case of the simplified Diamond model? This is largely an empirical matter.

Staying within the confines of the model, let us assume that the share of capital in total income \( \alpha \) is equal to 1/3, the annual population growth rate \( n \) is 1%, the annual rate of technical progress \( g \) is 2% and that the annual pure rate of time preference \( \rho \) is 2%. These are the empirical assumptions
that we have made so far. We also assume that the time period in the Diamond model is equal to 30 years. Thus, over thirty years, \( n = 0.348 \), \( g = 0.811 \) and \( \rho = 0.811 \).

Under these assumptions, the marginal product of steady state capital from (4.18) is equal to 4.44. The marginal product of the capital stock that corresponds to the golden rule is, from (4.19), equal to 1.44. Thus, based on these empirical estimates of the parameters, the possibility of dynamic inefficiency does not arise in the simplified Diamond model. With the assumptions about \( n \), \( g \) and \( \rho \), the share of capital should be lower than 1/6 instead of 1/3 for the possibility of dynamic inefficiency to arise.

In any case, when Abel, Mankiw, Summers and Zeckhauser (1989) explored the possibility of dynamic inefficiency empirically for the US economy and six other developed economies, they concluded that the marginal product of capital comfortably exceeds the long run growth rate, a finding that rules out the possibility of dynamic inefficiency for the developed economies.

On the basis of these observations, and the available empirical evidence, the possibility of dynamic inefficiency, although theoretically interesting, does not seem likely. Besides, this would mean that there is excess saving, which is in contrast with a number of other empirical features of developed market economies, such as current account deficits in many of them.

### 4.1.9 Dynamic Simulations of the Diamond Model

In order to investigate further the process of dynamic adjustment characterizing the simplified Diamond model, we can simulate, for specific values of the parameters of the model, the process of dynamic adjustment from one balanced growth path to another, when there is an exogenous permanent change in specific parameters. We shall concentrate on changes in the pure rate of time preference and total factor productivity.

The simulation assumes that the production function has the usual Cobb Douglas form,

\[
y_t = A k_t^\alpha
\]  

(4.2′)

In this case, and under the additional assumption that the inter-temporal elasticity of substitution is equal to unity, we simulate the accumulation equation (4.12).

Once one determines the dynamic path of capital per efficiency unit of labor, the corresponding path for output per efficiency unit of labor is derived from the production function (4.2).

The evolution of the real interest rate and real wages per efficiency unit of labor can be derived from equations (4.4α) and (4.4β), for a Cobb Douglas production function, and take the form,

\[
r_t = \alpha A k_t^{\alpha - 1}
\]  

(4.4a′)

\[
w_t = (1 - \alpha) A k_t^\alpha
\]  

(4.4b′)

Finally, the evolution of total private consumption can be derived from the consumption functions of young and old households.
We simulated the model for the following parameter values: $A=1$, $a=1/3$, $\theta=1$, $\rho=0.811$, $n=0.348$, $g=0.811$. Unlike the previous models, where we assumed that the time period is a calendar year, here we assume that the time period is thirty years, the half life of a generation. An annual rate of 1% is converted into a thirty-year rate of 34.8%, and an annual rate of 2% is converted into a thirty-year rate of 81.1%.

We consider two alternative scenarios. A permanent increase in the pure rate of time preference of the representative household $\rho$ by 5%, and a permanent increase in total factor productivity $A$ by 5%. The results of the simulations are presented in Figures 4.5 and 4.6.

In the simulation of Figure 4.5, the economy is on its original balanced growth path, and in period 1 the pure rate of time preference of the young generation $\rho$ rises by 5%, from 0.811 to 0.852, where it remains thereafter. This increase leads directly to an increase in consumption, a decline in the savings rate, a gradual de-cumulation of capital, a gradual decline in output and real wages and a gradual increase in the real interest rate. The reason for the decline in real wages is the gradual reduction of the marginal product of labor, while the reason for the increasing real interest rate is the gradual increase in the marginal product of capital. Both are caused by the declining capital stock. The economy gradually converges to a new balanced growth path. In this new path, capital per efficiency unit of labor is lower by about 2.2% compared to the original path, output and real wages are lower by 0.7%, consumption is lower by 0.2% (due to the decline in the saving rate), while the real interest rate has increased by 1.4%. The savings rate for the economy as a whole, which is endogenous in this model, has declined from 24.7% to 24.4%.

In the simulation of Figure 4.6 the economy is on its original balanced growth path, and in period 1 total factor productivity $A$ rises permanently by 5%, from 1 to 1.05. This increase leads immediately to an increase in production, consumption, savings and the marginal product of both the labor (real wage) and capital (real interest rate). The increase in savings causes a gradual accumulation of capital, which leads to a further gradual increase in production, consumption and the real wage, and a gradual decline in the real interest rate. The reason for the declining real interest rate is the gradual reduction of the marginal product of capital caused by the accumulation of capital. The economy gradually converges to a new balanced growth path. In this new path capital per efficiency unit of labor is increased by about 7.7%, output, consumption and real wages have also increased by 7.7%, while the real interest rate has returned to its original equilibrium. The equilibrium real interest rate only depends on the pure rate of time preference of the representative household, the inter-temporal elasticity of substitution and the rate of technical progress. The reason why an increase in productivity by 5% leads to an increase in real output by 7.7%, i.e more than 5%, is that higher total productivity causes a temporary increase in the savings rate and capital accumulation, which causes secondary increases in the capital stock, real output, consumption and real wages.

As we see from these simulations, the dynamic behavior of the simplified Diamond model of overlapping generations is qualitatively similar to the dynamic behavior of the Ramsey model of a representative household. Although the models differ in the specific assumptions they make about the determination of aggregate savings, they result in qualitatively similar properties regarding the process of adjustment towards the balanced growth path. In addition, despite the prediction that the savings rate is not constant along the adjustment path, as assumed in the Solow model, in all other respects, the predictions of the Diamond and Ramsey models about the adjustment path are very similar to the predictions of the Solow model.
4.2 The Blanchard and Weil Model of Perpetual Youth

We now turn to an alternative overlapping generations model, the model of *perpetual youth* of Blanchard and Weil.\(^4\)

In the Blanchard-Weil model it is assumed that new households are being born continuously, with each household having an infinite time horizon. All generations of households supply labor and have the same efficiency of labor, regardless of their date of birth. Consequently, all households have the same labor productivity and present value of labor income and the same (infinite) time horizon. For this reason this model is often referred to as the model of perpetual youth.\(^5\)

In the original version of this model by Blanchard (1985), at any instant there is a constant probability of death, which is independent of the age of households. In the model of Weil (1989), the probability of death is zero, i.e. households not only have an infinite time horizon but discount the future only by using the pure rate of time preference and not the probability of death.

What emerges from the model of perpetual youth is that the main differences of the predictions of overlapping generations models from representative household models result from differences in the age of different households, and not the possibility of death.\(^6\)

Technology and market structure in this model is similar to the previous models we have analyzed so far. We have competitive markets and a neoclassical production function with constant returns to scale.

4.2.1 Definitions

The definitions of the variables are analogous to the definitions we have used so far. As with the Solow, Ramsey and Diamond models, we shall focus on the following set of *endogenous variables*:

- \(Y\): Aggregate Output (or \(y\), Output per efficiency unit of labor)
- \(K\): Aggregate (physical) Capital Stock (or \(k\), Capital per efficiency unit of labor)
- \(C\): Aggregate Consumption (or \(c\), Consumption per efficiency unit of labor)
- \(r\): Real interest rate
- \(wh\): Real wage per worker (or \(w\), Real Wage per efficiency unit of labor)

The *exogenous variables* and *exogenous parameters* in the model are defined as follows:

- \(t\): time (a continuous exogenous variable)
- \(L\): Total Population (an exogenous variable that depends on time)

\(^4\)This model was first presented by Blanchard (1985), and was based to some extent on the model of Yaari (1965).

\(^5\)There are variations of this model in which productivity and labor income is a negative function of the age of the household. The properties of these variants of the model are closer to some of the properties of the Diamond model, in the sense that the possibility of dynamic inefficiency may arise (see Blanchard and Fischer 1989).

\(^6\)In the perpetual youth model, accumulated savings depend on the date of birth of households. This causes differences in the savings behavior of households depending on their date of birth, because households of different generations have different accumulated savings. This results in direct effects of the aggregate capital stock (and other assets such as government bonds) on the behavior of aggregate consumption (see Buiter 1988).
h  “efficiency” of labor (an exogenous variable that depends on time)

n  rate of growth of population (exogenous parameter)

g  rate of technical progress (exogenous parameter)

δ  rate of depreciation of capital (exogenous parameter)

ρ  pure rate of time preference of households (exogenous parameter)

With regard to L and h we assume that \[ \lim_{t \to \pm \infty} (L(t)) = 1, \lim_{t \to \pm \infty} (h(t)) = 1. \]

### 4.2.2 The Production Function

Production takes place on the basis of technological possibilities described by a neoclassical production function, similar to the one assumed in the Solow, Ramsey and Diamond models.

\[ Y(t) = F(K(t), h(t)L(t)) \]  \hspace{1cm} (4.20)

The production function is characterized by constant returns to scale, so it can be written as,

\[ y(t) = f(k(t)) \]  \hspace{1cm} (4.21)

where,

\[ y = Y/hL \quad \text{Output per efficiency unit of labor} \]

\[ k = K/hL \quad \text{Capital stock per efficiency unit of labor} \]

\[ f(k) = F(k, l) \quad \text{Production function per efficiency unit of labor} \]

### 4.2.3 The Inter-temporal Utility Function of Households and Aggregate Consumption

Following Weil (1989), we will assume that households have an infinite time horizon (i.e they are dynasties), but differ in their birth date. Each instant, nL new households are born. New households have no connection with old households whatsoever. The typical household born at time \( j \) chooses its path of current and future consumption in order to maximize the inter-temporal utility function,

\[ U_j = \int_{t=j}^{\infty} e^{-\rho t} u(c(j,t)) dt \]  \hspace{1cm} (4.22)

where,

\[ c(j,t) \quad \text{consumption of household } j \text{ at instant } t \]

\[ u \quad \text{instantaneous utility function} \]

\[ \rho \quad \text{pure rate of time preference} \]

Following Blanchard (1985) and Weil (1989) we shall assume that the instantaneous utility function is logarithmic. This is equivalent to assuming a unitary elasticity of inter-temporal substitution of consumption,

\[ u(c(j,t)) = \ln c(j,t) \]  \hspace{1cm} (4.23)
All households have a constant number of members (assumed to be equal to one), with each supplying one unit of labor. All households, irrespective of their date of birth, can borrow and lend freely at the market determined real interest rate \( r \).

Moreover, we assume that there is a large number of competitive firms, each with a production function similar to (4.20). Markets are competitive and therefore the real interest rate and the real wage defined by,

\begin{align*}
  r(t) &= f'(k(t)) \\  w(t) &= f(k(t)) - k(t)f'(k(t))
\end{align*}

where \( f'(k) \) is the marginal product of capital. We have assumed that the depreciation rate \( \delta \) is equal to zero.

Therefore, household \( j \) maximizes (4.22) under the constraint,

\[ k(j,t) = r(t)k(j,t) + w(t)h(t) - c(j,t) \quad (4.26) \]

where \( k(j,j) = 0 \).

From the first order conditions for a maximum, we can derive Euler equation for consumption of household \( j \).

\[ c(j,t) = (r(t) - \rho)c(j,t) \quad (4.27) \]

Integrating (4.27) over time, and using the fact that the present value of consumption is equal to initial assets plus the present value of labor income for all households, we get the consumption function,

\[ c(j,t) = \rho (k(j,t) + W(j,t)) \quad (4.28) \]

where,

\[ W(j,t)\int_{\nu}^{\tau} e^{-\int_{\nu}^{\tau} r(v)dv} w(v)h(v)dv = W(t) \quad (4.29) \]

is the present value at time \( t \) of labor income (human capital). From (4.29) we can confirm that the present value of labor income is the same for all households, irrespective of their birth date, because the real wage is the same for all households, as is labor efficiency, and all households supply one unit of labor per instant.

From (4.28) we can see that the average and marginal propensity to consume out of wealth (the sum of non-human and human capital) is equal to the pure rate of time preference. This is a consequence of logarithmic preferences and the assumption that there is no population growth within households.
The aggregate variables $C(t)$, $K(t)$ and $W(t)$ will be determined by the integral of the variables that correspond to every household at time $t$, i.e. for $j \leq t$. For every generation $j$ its size, equal to $n e^{\eta j}$, is taken into account. Obviously, younger generations are more numerous than older generations.

As a result, aggregate consumption and aggregate physical and human capital are given by,

\begin{align}
C(t) &= \int_{j=-\infty}^{t} ne^{\eta j} c(j,t) \, dj \tag{4.30a} \\
K(t) &= \int_{j=-\infty}^{t} ne^{\eta j} k(j,t) \, dj \tag{4.30b} \\
W(t) &= \int_{j=-\infty}^{t} ne^{\eta j} W(j,t) \, dj = e^{\eta t} \bar{W}(t) \tag{4.30c}
\end{align}

From (4.28) and (4.30a) it follows that,

\begin{align}
C(t) &= \rho (K(t) + W(t)) \tag{4.31}
\end{align}

From (4.31),

\begin{align}
\dot{C}(t) &= \rho \left( \dot{K}(t) + \dot{W}(t) \right) \tag{4.32}
\end{align}

From (4.26) and (4.30b),

\begin{align}
\dot{K}(t) &= r(t) K(t) + w(t) h(t) e^{\eta t} - C(t) \tag{4.33}
\end{align}

From (4.30c),

\begin{align}
\dot{W}(t) &= (r(t) + n) W(t) - w(t) h(t) e^{\eta t} \tag{4.34}
\end{align}

Finally, from (4.31) to (4.34),

\begin{align}
\dot{C}(t) &= (r(t) + n - \rho) C(t) - n \rho K(t) \tag{4.35}
\end{align}

Consumption and the capital stock per efficiency unit of labor are defined as,

\begin{align}
c(t) &= \frac{C(t)}{h(t)L(t)} = C(t) e^{-(n+\rho) t} \tag{4.36} \\
k(t) &= \frac{K(t)}{h(t)L(t)} = K(t) e^{-(n+\rho) t} \tag{4.37}
\end{align}

From (4.35) and (4.36) it follows that,
From (4.24) the real interest rate is equal to the marginal product of capital. (4.38’) can thus be written as,

\[ c(t) = [r(t) - \rho - g]c(t) - n\rho k(t) \]  (4.38’)

From (4.33) and (4.37), using the properties of the production function and the fact that the marginal product of capital is equal to the real interest rate and the marginal product of labor is equal to the real wage, we get the following accumulation equation for capital. This is the usual accumulation equation which defines the change in the capital stock per efficiency unit of labor as the difference between savings and equilibrium investment \((n+g)k\).

\[ k(t) = f(k(t)) - c(t) - (n + g)k(t) \]  (4.39)

We can now use (4.38) and (4.39) to analyze both the balanced growth path and the process of adjustment towards the balanced growth path.

### 4.2.4 Dynamic Adjustment towards the Balanced Growth Path

We shall analyze the balanced growth path and the adjustment path using a phase diagram analogous to the phase diagram we used for the Ramsey model.

On the balanced growth path, we shall have that,

\[ c(t) = k(t) = 0 \]

From (4.38), the geometric locus for which consumption per efficiency unit of labor is constant, is defined as,

\[ c(t) = \frac{n\rho k(t)}{f'(k(t)) - \rho - g} \]  (4.40)

From (4.39), the geometric locus for which capital per efficiency unit of labor is constant, is defined by,

\[ c(t) = f(k(t)) - (n + g)k(t) \]  (4.41)

The steady state (balanced growth path) is determined at the intersection of these two geometric loci, i.e when (4.40) and (4.41) are simultaneously satisfied.

The dynamic analysis of the steady state in this model is presented in Figure 4.5. The steady state is determined at the point \((k^*, c^*)\), where (4.40) and (4.41) are simultaneously satisfied.
It is worth noting that this equilibrium lies to the left of the equilibrium that would prevail in the case of a representative household. In the steady state of the model of overlapping generations with perpetual youth, both the capital stock and the level of consumption (per efficiency unit of labor) are lower than the values corresponding to the modified golden rule. This is because current generations do not take into account the welfare of future generations, and end up saving less than a representative household that takes the welfare of future generations into account. For this reason, and despite competitive markets, the equilibrium in the model of perpetual youth is not socially efficient, in the sense of maximizing social welfare. On the other hand, the possibility of dynamic inefficiency does not arise in this model. The capital stock cannot be higher than the level corresponding to the golden rule.

As in the Ramsey model there is a unique saddle path leading to the steady state. As consumption is non predetermined variable and the capital stock a predetermined variable, from any initial position, consumption will directly adjust to put the economy on the unique saddle path leading to the steady state. To the left of $k^*$, consumption is lower than its steady state value, and the economy accumulates capital. During this process, capital and consumption per efficiency unit of labor are increasing. The opposite occurs if the initial capital stock is to the right of the steady state capital stock. Both the capital stock and consumption per efficiency unit of labor are higher than in the steady state and are decreasing on the unique saddle path.

In Figure 4.6, we analyze the effects of an increase in the pure rate of time preference of households.

An increase in pure rate of time preference leads to a shift of the steady state consumption function to the left. In the new equilibrium $(c^{**}, k^{**})$ both consumption and the capital stock per efficiency unit of labor will be lower. When the shift occurs, consumption increases and puts the economy on the saddle path corresponding to the new balanced growth path. Savings are lower than equilibrium investment, and the economy starts to de-cumulate capital, gradually adapting towards the new balanced growth path, with lower consumption and capital per efficiency unit of labor.

A decrease in the pure rate of time preference has the opposite effect. In the new equilibrium both consumption and capital will be higher. When the shift occurs, consumption decreases, the economy begins to accumulate additional capital and the economy gradually adjusts to the new balanced growth path with higher capital and consumption per efficiency unit of labor.

We thus see that the pure rate of time preference is in this model a key factor determining savings behavior and the balanced growth path. The higher it is, the less households of all generations save, as they prefer more utility from consumption today relative to utility from future consumption.

### 4.2.5 Dynamic Simulations of the Blanchard Weil Model

In order to investigate further the dynamic properties of the Blanchard Weil model, we can simulate, for specific values of the parameters, the process of dynamic adjustment from one balanced growth path to another, when there is an exogenous permanent change in one specific parameter, such as the pure rate of time preference or total factor productivity.

In order to simulate the model we shall convert it from a continuous time model to a discrete time model (see Annex to Chapter 3).
The pair of equations we use in the simulation is (A4.17) και (A4.18), for a Cobb Douglas production function and a positive depreciation rate \( \delta \). These take the form,

\[
k_{t+1} = \frac{1}{(1+n)(1+g)} \left( Ak_t^\alpha + (1-\delta)k_t - c_t \right)
\]

\[
c_{t+1} = \frac{1+\alpha Ak_t^{\alpha-1} - \delta}{(1+\rho)(1+g)} c_t - \frac{n\rho}{(1+n)(1+g)(1+\rho)} k_t
\]

After we determine the adjustment path of capital and private consumption per efficiency unit of labor, the path of real output can be calculated from the production function, and the paths of the real interest rate and the real wage from the marginal productivity conditions for capital and labor.

The parameter values we use in the simulations are the following: \( A=1 \), \( \alpha=0.333 \), \( \rho=0.02 \), \( n=0.01 \), \( g=0.02 \), \( \delta=0.03 \). These values are the same as the ones we used for the Ramsey model in Chapter 3, and consistent with the values we used for the Solow model in Chapter 2 and the Diamond model in the first part of this chapter.

We consider two alternative scenarios. A permanent increase in the pure rate of time preference of the households \( \rho \) by 5%, from 0.02 to 0.021, and a permanent increase in total factor productivity \( A \) by 5%, from 1 to 1.05. The results of the simulations are presented in Figures 4.9 and 4.10.

In the simulation of Figure 4.9, the economy is on its original balanced growth path, and in period 1 the pure rate of time preference of households \( \rho \) rises permanently and unexpectedly by 5%, from 0.02 to 0.021. This increase leads directly to an increase in consumption, a decline in the savings rate, a gradual de-cumulation of physical capital, a gradual decline in output, a gradual decline in real wages and a gradual increase in the real interest rate. The reason for declining real wages is the gradual reduction of the marginal product of labor caused by the falling capital stock, while the reason that the increasing real interest rate is that the marginal product of capital gradually increases because of the de-cumulation of capital. The economy gradually converges to a new balanced growth path. On the new balanced growth path, capital per efficiency unit of labor is lower by around 2.1%, output, and real wages by 0.7%, consumption by 0.2% (due to the decline in the saving rate), while the real interest rate has increased by 0.1 percentage points, the same as the increase in the pure rate of time preference. The steady state savings rate, which is endogenous in this model, falls, from 28.0% to 27.6%.

In the simulation of Figure 4.10, the economy is initially on its original balanced growth path. In period 1 total factor productivity \( A \) rates rises permanently and unexpectedly by 5%, from 1 to 1.05. This increase leads to an immediate increase in production, consumption, savings and the marginal product of both labor (the real wage) and capital (the real interest rate). The increase in savings causes a gradual accumulation of capital, which leads to a further gradual increase in production and consumption, a further gradual increase in the real wage, but a gradual fall in real interest rates. The reason for the falling real interest rate is the gradual reduction of the marginal product of capital caused by the accumulation of capital. The economy gradually converges to a new balanced growth path. In this capital per efficiency unit of labor has increased by about 7.4%, output, consumption and real wages are also higher by 7.6%, while the real interest rate has returned to its original equilibrium. The steady state real interest rate only depends on the pure rate of time
preference of the representative household, the inter-temporal elasticity of substitution and the rate of technical progress. The reason why an increase in total factor productivity by 5% leads to an increase in real income by 7.6%, i.e. more than 5%, is that productivity growth causes a temporary increase in the savings rate and capital accumulation, which in turn causes additional induced increases in real incomes and consumption.

The dynamic behavior of the perpetual youth, overlapping generations, model of Blanchard and Weil is similar to the dynamic behavior of the representative household model and the Diamond model of overlapping generations.

However, it is worth noting that, as the theoretical analysis suggests, the savings rate in the overlapping generations model is lower than the savings rate in the representative household model. For exactly the same parameter values, the savings rate in the perpetual youth model is 28% on the original balanced growth path, while the corresponding savings rate in the representative household model of Chapter 2 is equal to 28.5%.

The higher savings rate in the representative household model has as a result a higher steady state capital stock, real output, consumption and real wages in this model, compared to the perpetual youth model of overlapping generations. On the other hand, the steady state real interest rate is lower than in the perpetual youth model.

For the initial values of the parameters used in the simulations, capital per efficiency unit of labor on the balanced growth path is equal to 10.275 in the representative household model and only 10.002 in the perpetual youth model (higher by 2.73%). Real income per efficiency unit of labor is equal to 2.172 in the representative household model, compared with 2.153 in the perpetual youth model (higher by 0.9%). The same applies to steady state real wages, which are higher by 0.9% in the representative household model. The difference in consumption is 0.2% for the model of the representative household, because of the higher savings rate. Finally, due to the lower capital stock per efficiency unit of labor, the real interest rate on the balanced growth path is 4.2% in the perpetual youth model, compared with 4% in the corresponding representative household model.

As we see from these simulations, the dynamic behavior of the Blanchard Weil model of overlapping generations is qualitatively similar to the dynamic behavior of the Ramsey and Diamond models. In addition, despite the prediction that the savings rate is not constant along the adjustment path, as assumed in the Solow model, in all other respects, the predictions of the Blanchard Weil model about the adjustment path are very similar to the predictions of the Solow model.

4.3 Conclusions

Aggregate savings behavior in overlapping generations models is not characterized by social efficiency, as in the representative household model. Besides in economies where there is no representative household, comparing welfare across different households is largely arbitrary.

In any case, although the dynamic behavior of models of overlapping generations is largely similar to the dynamic behavior of the representative household model, there is no guarantee of efficiency in overlapping generations models. Thus, in these models there is scope for policy interventions that would result in greater social efficiency as the competitive equilibrium does not necessarily lead to
optimal results. Furthermore, in the Diamond model, there is the theoretical possibility of dynamic inefficiency, although empirical considerations suggest that this possibility is unlikely.

Because the competitive equilibrium does not result in an optimal savings behavior in overlapping generations models, such models also have different properties than the representative household model, regarding both the short run and the long run effects of budgetary (fiscal) policies and monetary policies that affect the rate of growth of the money supply. These are questions that we shall investigate in the next two chapters.
Annex to Chapter 4:  
The Blanchard Weil Model in Discrete Time

In this annex we present the Blanchard Weil model of perpetual youth in discrete time. Instead of treating time as a continuous variable, we treat it as consecutive time periods, where \( t=0,1,2,... \). Variable \( x_t \) denotes variable \( x \) in period \( t \).

Population and the efficiency of labor grow at corresponding rates \( n \) and \( g \) per period. We therefore have,

\[
L_t = L_0 (1+n)^t \\
h_t = h_0 (1+g)^t
\]  
(A4.1)  
(A4.2)

The inter-temporal utility function of the household born in period \( j \) is given by,

\[
U(j) = \sum_{t=j}^{\infty} \left( \frac{1}{1+\rho} \right)^{t-j} u(c(j)_t)
\]  
(A4.3)

where \( \rho \) is the pure rate of time preference of the household, \( u \) the per period utility function, and \( c(j)_t \) the consumption of the household born in period \( j \).

We assume, as in the continuous time case, that the utility function \( u \) is logarithmic.

\[
u(c(j)_t) = \ln c(j)_t
\]  
(A4.4)

Under this hypothesis, the household born in period \( j \) maximizes its inter-temporal utility function,

\[
U(j) = \sum_{t=j}^{\infty} \left( \frac{1}{1+\rho} \right)^{t-j} \ln c(j)_t
\]  
(A4.5)

under the constraint of accumulation of assets,\(^7\)

\[
k(j)_{t+1} = (1+r_t) k(j)_t + w_t h_t - c(j)_t
\]  
(A4.6)

The Lagrangean for the maximization of (A4.5) under the constraint (A4.6) is given by,

\[
U(j) = \sum_{t=j}^{\infty} \left( \frac{1}{1+\rho} \right)^{t-j} \left[ \ln c(j)_t + \lambda_t \left( (1+r_t) k(j)_t + w_t h_t - c(j)_t - k(j)_{t+1} \right) \right]
\]  
(A4.7)

where \( \lambda_t \) is the Lagrange multiplier for period \( t \).

\(^7\) We shall assume in this analysis that the depreciation rate of the capital stock \( \delta \) is equal to zero.
The first order conditions for consumption and the capital stock are given by,

\[ \lambda_t = \frac{1}{c(j)} \]  
\[ \lambda_t = \frac{1 + r_{t+1}}{(1 + \rho)} \lambda_{t+1} \]  

Combining (A4.8) and (A4.9) we get,

\[ \frac{c(j)_{t+1}}{c(j)_t} = \frac{1 + r_{t+1}}{(1 + \rho)} \]  

(A4.10) is the Euler equation for consumption, of the household born in period \( j \).

The size of the generation born in period \( j \) is given by,

\[ L_j = nL_0(1 + n)^{j-1} \]  

for \( j > 1 \). \( L_0 \) is the size of population (number of households) in period 0.

Aggregating the restriction (A4.6) and the Euler equation (A4.10), taking into account the size of each generation as given by (A4.11), after expressing all variables per efficiency unit of labor, we get that,

\[ k_{t+1} = \frac{1}{(1 + n)(1 + g)} ((1 + r)k_t + w_t - c_t) \]  

(A4.12)

\[ c_{t+1} = \frac{1 + r_{t+1}}{(1 + \rho)(1 + g)} c_t - \frac{np}{(1 + n)(1 + g)(1 + \rho)} k_t \]  

(A4.13)

It is worth noting that (A4.12) is exactly the same as the accumulation equation in the corresponding representative household model (see Annex to Chapter 2). The equation for aggregate consumption has the form of (A2.10) for \( \theta = 1 \), but it also depends negatively on an additional term involving the capital stock per efficiency unit of labor, because in the representative household model new generations enter the economy with zero capital stock.

Assuming the neoclassical production function and competitive labor and capital markets,

\[ y_t = f(k_t) \]  
\[ r_t = f'(k_t) \]  
\[ w_t = f(k_t) - f'(k_t)k_t \]  

(A4.14) (A4.15) (A4.16)

Substituting (A4.14), (A4.15) and (A4.16) in (A4.12) and (A4.13) we get,
The second order system of (non linear) difference equations (A4.17) και (A4.18) describes the dynamic adjustment of the perpetual youth overlapping generations model in discrete time.

On the balanced growth path, \( c_{t+1} = c_t = c^* \) and \( k_{t+1} = k_t = k^* \). From (A4.17) and (A4.18) it follows that,

\[
k_{t+1} = \frac{1}{(1+n)(1+g)} (k_t + f(k_t) - c_t)
\]

\[
c_{t+1} = \frac{1 + f'(k_{t+1})}{(1+\rho)(1+g)} c_t - \frac{n\rho}{(1+n)(1+g)(1+\rho)} k_t
\]

(A4.19) and (A4.20) determine consumption and the capital stock per efficiency unit of labor on the balanced growth path.

For positive consumption we must have that,

\[
(1 + f'(k^*)) > (1 + \rho)(1 + g)
\]

(A4.21)

Consequently, the steady state capital per efficiency unit of labor is lower than the capital stock that would correspond to the representative household model in which the rate of entry of new households in the economy \( n \) equals zero, and which would imply that ,

\[
(1 + f'(k^*)) = (1 + \rho)(1 + g)
\]

(A4.22)

As mentioned in our analysis in continuous time, in the overlapping generations model of perpetual youth, the steady state capital stock is lower than the one corresponding to the modified golden rule.
Figure 4.1
Multiple Equilibria in the Diamond Model
Figure 4.2
The Simplified Diamond Model
with Logarithmic Preferences and a Cobb Douglas Production Function
Figure 4.3
An Increase in the Pure Rate of Time Preference
in the Simplified Diamond Model
Figure 4.4
An Increase in Total Factor Productivity in the Simplified Diamond Model
Figure 4.5
A Dynamic Simulation of an Increase in the Pure Rate of Time Preference by 5% in the Simplified Diamond Model
Figure 4.6
A Dynamic Simulation of an Increase in Total Factor Productivity by 5% in the Simplified Diamond Model
Figure 4.7
Steady State Equilibrium and Dynamic Adjustment in the Blanchard Weil Model
Figure 4.8
Short Run and Long Run Effects of a Rise in the Pure Rate of Time Preference in the Blanchard Weil Model
Figure 4.9
A Dynamic Simulation of an Increase in the Pure Rate of Time Preference by 5% in the Blanchard Weil Model
Figure 4.10
A Dynamic Simulation of an Increase in Total Factor Productivity by 5% in the Blanchard Weil Model
References


